The electromagneto-acoustic surface wave in a piezoelectric medium: The Bleustein–Gulyaev mode

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By discarding the quasi-static approximation, this paper gives the exact solutions of a shear horizontal electromagneto-acoustic surface wave mode in a class of piezoelectric media. As the wave speed is much less than the speed of light, the solution degenerates to the well-known Bleustein–Gulyaev wave, or Maerfeld–Tournois wave. Taking into account both optical effect as well as the contribution from the rotational part of electric field, the solutions obtained here are not only valid for any wave speed range, but also provide accurate formulas to evaluate the acousto-optic interaction due to the piezoelectricity. © *1996 American Institute of Physics.* [S0021-8979(96)06121-X]

I. INTRODUCTION

About thirty years ago, Bleustein[1] and Gulyaev[2] simultaneously discovered that there exists a shear horizontal (SH) electro-acoustic surface mode in a class of transversely isotropic piezoelectric media, which is known today as the Bleustein–Gulyaev wave. The Bleustein–Gulyaev (BG) surface wave is an unique result in the repertoire of surface acoustic wave (SAW) theory, because it has no counterpart in purely elastic solids. As a matter of fact, since then, the BG wave theory has become one of the cornerstones for the modern electro-acoustic technology.

The BG wave is essentially a coupled surface wave between the acoustic mode and the soft ferroelectric mode; in other words, the quasi-static approximation is adopted for the electromagnetic field. Under this assumption, both the optical effect as well as the contribution from the rotational part of electric field are neglected. Although it is generally believed that the optical effect is minor, it is certainly of practical interest to accurately predict the piezoelectricity-induced electromagnetic radiation, which might be helpful in some engineering applications, such as optical detection, as well as nondestructive evaluation in general. In this paper, a detailed account is given of the coupled SH electromagneto-acoustic surface wave in a transversely isotropic piezoelectric medium.

In early studies, besides technical considerations, the main reason for adopting the quasi-static approximation might have been, perhaps, a psychological one. The perception was that solving a fully-coupled Maxwell–Christoffel equation might be too involved, or too complicated to obtain any meaningful results in physics. In this paper, it has been shown, on the contrary, that there exist some remarkable simple velocity equations for the fully-coupled SH electromagneto-acoustic surface wave.

II. FORMULATION

By adopting the notations in Auld,[3,4] the governing equations of the problem are listed as follows.

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(i) Maxwell's equations

$$-\nabla\times\mathbf{E}=\frac{\partial\mathbf{B}}{\partial t},\tag{1}$$

$$\nabla\times\mathbf{H}=\frac{\partial\mathbf{D}}{\partial t},\tag{2}$$

where $\mathbf{E},\mathbf{B}$, and $\mathbf{H}$ are the electric field, magnetic induction, and magnetic field respectively;

(ii) Equations of motion

$$\nabla\cdot\boldsymbol{\sigma}=\rho\frac{\partial^2\mathbf{u}}{\partial t^2}-\mathbf{F},\tag{3}$$

where $\boldsymbol{\sigma}$ is the stress tensor, $\mathbf{u}$ is the displacement vector, and $\mathbf{F}$ is the body force;

(iii) The constitutive equations

$$\mathbf{B}=\mu_0\mathbf{H},\tag{4}$$

$$\mathbf{D}=\boldsymbol{\epsilon}^s\cdot\mathbf{E}+\mathbf{e}\mathbf{:}\boldsymbol{\varepsilon},\tag{5}$$

$$\boldsymbol{\sigma}=-\mathbf{e}\cdot\mathbf{E}+\mathbf{c}^E\mathbf{:}\boldsymbol{\varepsilon},\tag{6}$$

where $\boldsymbol{\epsilon}^s,\mathbf{e}$, and $\mathbf{c}^E$ are the specific dielectric tensor, piezoelectric constant tensor, and elastic stiffness constant tensor respectively, and μ_0 is the magnetic permeability constant in the vacuum.

In the constitutive equations (5) and (6), the strain tensor $\boldsymbol{\varepsilon}$ is defined as

$$\boldsymbol{\varepsilon}:=\tfrac{1}{2}[\nabla\mathbf{u}+(\nabla\mathbf{u})^T]=:\nabla_s\mathbf{u}.\tag{7}$$

Letting $\mathbf{F}=0$, one may derive the following fully-coupled Maxwell–Christoffel equations [Auld[4] (8.105) and (8.106)] by proper manipulations,

$$\nabla\cdot\mathbf{c}^E\mathbf{:}\nabla_s\mathbf{u}=\rho\frac{\partial^2\mathbf{u}}{\partial t^2}+\nabla\cdot(\mathbf{e}\cdot\mathbf{E}),\tag{8}$$

$$-\nabla\times\nabla\times\mathbf{E}=\mu_0\boldsymbol{\epsilon}^s\cdot\frac{\partial^2\mathbf{E}}{\partial t^2}+\mu_0\mathbf{e}\mathbf{:}\nabla_s\frac{\partial^2\mathbf{u}}{\partial t^2}.\tag{9}$$

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Consider the coupling between the anti-plane acoustic mode and the in-plane electromagnetic mode, i.e.,

$$\mathbf{u}=[0,0,w(x_1,x_2,t)] \tag{10}$$

$$\mathbf{E}=[E_1(x_1,x_2,t),E_2(x_1,x_2,t),0]. \tag{11}$$

The coupled wave equations (8) and (9) can be simplified drastically. For the hexagonal (6 mm) piezoelectric material, equations (8) and (9) reduce to

$$c_{44}^E\nabla^2 w=\rho\frac{\partial^2 w}{\partial t^2}+e_{15}\nabla\cdot\mathbf{E}, \tag{12}$$

$$-\nabla\times\nabla\times\mathbf{E}=\mu_0\epsilon_{11}^s\frac{\partial^2\mathbf{E}}{\partial t^2}+\mu_0 e_{15}\nabla\frac{\partial^2 w}{\partial t^2}, \tag{13}$$

where the differential operator ∇ and the electric field $\mathbf{E}$ are redefined as two-dimensional vectors, i.e.,

$$\nabla:=\mathbf{i}\frac{\partial}{\partial x_1}+\mathbf{j}\frac{\partial}{\partial x_2} \tag{14}$$

$$\mathbf{E}:=E_1\mathbf{i}+E_2\mathbf{j}. \tag{15}$$

Let

$$\mathbf{E}=-\nabla\phi-\frac{1}{c_\ell}\frac{\partial\mathbf{A}}{\partial t}, \tag{16}$$

where ϕ and $\mathbf{A}$ are the scalar potential and the vector potential respectively, and the constant, $c_\ell:=(\mu_0\epsilon_{11}^s)^{-1/2}$, is the speed of light in the piezoelectric material. The decomposition (16) can be uniquely determined by imposing the following Lorentz gauge constraint within the transversely isotropic plane,

$$\nabla\cdot\mathbf{A}+\frac{1}{c_\ell}\frac{\partial\phi}{\partial t}=0. \tag{17}$$

Subsequently, the coupled wave equations (12) and (13) can be further separated into two groups, namely, the purely electro-acoustic wave equations,

$$\begin{cases} c_{44}^E\nabla^2 w-\rho\dfrac{\partial^2 w}{\partial t^2} = -e_{15}\left(\nabla^2\phi-\dfrac{1}{c_\ell}\dfrac{\partial^2\phi}{\partial t^2}\right), \\ \dfrac{e_{15}}{\epsilon_{11}^s}\nabla^2 w = \left(\nabla^2\phi-\dfrac{1}{c_\ell}\dfrac{\partial^2\phi}{\partial t^2}\right); \end{cases} \tag{18}$$

and the rotational part of electromagneto-acoustic wave equations,

$$\begin{cases} \nabla^2 A_1-\dfrac{1}{c_\ell^2}\dfrac{\partial^2 A_1}{\partial t^2} = -\mu_0 e_{15}c_\ell\dfrac{\partial^2 w}{\partial t\partial x_1}, \\ \nabla^2 A_2-\dfrac{1}{c_\ell^2}\dfrac{\partial^2 A_2}{\partial t^2} = -\mu_0 e_{15}c_\ell\dfrac{\partial^2 w}{\partial t\partial x_2}. \end{cases} \tag{19}$$

Define

$$\overline{c_{44}}:=c_{44}^E+\frac{e_{15}^2}{\epsilon_{11}^s}, \tag{20}$$

$$c_a:=\left(\frac{\overline{c_{44}}}{\rho}\right)^{1/2}, \tag{21}$$

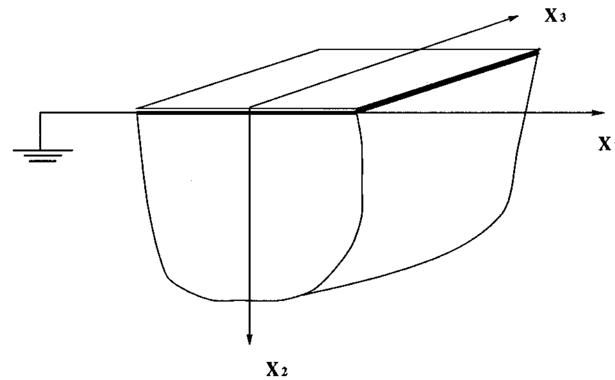

FIG. 1. The piezoelectric half space covered with a layer of conducting film.

$$f:=\frac{c_\ell^2}{c_\ell^2-c_a^2}; \tag{22}$$

and introduce a new scalar potential function ψ,

$$\psi:=\phi-\frac{e_{15}}{\epsilon_{11}^s}fw. \tag{23}$$

The purely electro-acoustic wave equations (18) can be completely decoupled as follows,

$$\begin{cases} \nabla^2 w-\dfrac{1}{c_a^2}\dfrac{\partial^2 w}{\partial t^2} = 0, \\ \nabla^2\psi-\dfrac{1}{c_\ell^2}\dfrac{\partial^2\psi}{\partial t^2} = 0. \end{cases} \tag{24}$$

Accordingly, the relevant constitutive equations take the following forms,

$$\sigma_{13}=\tilde{c}_{44}\frac{\partial w}{\partial x_1}+e_{15}\frac{\partial\psi}{\partial x_1}+\frac{e_{15}}{c_\ell}\frac{\partial A_1}{\partial t}, \tag{25}$$

$$\sigma_{23}=\tilde{c}_{44}\frac{\partial w}{\partial x_2}+e_{15}\frac{\partial\psi}{\partial x_2}+\frac{e_{15}}{c_\ell}\frac{\partial A_2}{\partial t}, \tag{26}$$

$$D_1=e_{15}(1-f)\frac{\partial w}{\partial x_1}-\epsilon_{11}^s\frac{\partial\psi}{\partial x_1}-\frac{\epsilon_{11}^s}{c_\ell}\frac{\partial A_1}{\partial t}, \tag{27}$$

$$D_2=e_{15}(1-f)\frac{\partial w}{\partial x_2}-\epsilon_{11}^s\frac{\partial\psi}{\partial x_2}-\frac{\epsilon_{11}^s}{c_\ell}\frac{\partial A_2}{\partial t}, \tag{28}$$

where $\tilde{c}_{44}:=c_{44}^E+fe_{15}^2/\epsilon_{11}^s$.

III. SOLUTIONS

In what follows, the SH electromagneto-acoustic wave propagation problem discussed above is examined under three different sets of boundary conditions.

Problem 1 (The grounded surface solution). In this problem, the surface of the piezoelectric half space (see Fig. 1) is covered with an infinitesimally thin perfect conducting film, which implies that the additional mechanical effect is neglected, and the scalar potential of the electric field is set to be zero on the surface. This group of boundary conditions are expressed as follows,

$$\sigma_{23}(x_1,0,t)=\tilde{c}_{44}\frac{\partial w}{\partial x_2}+e_{15}\frac{\partial \psi}{\partial x_2}+\frac{e_{15}}{c_\ell}\frac{\partial A_2}{\partial t}=0, \tag{29}$$

$$\phi(x_1,0,t)=\psi+\frac{e_{15}}{\epsilon_{11}^s}fw=0. \tag{30}$$

Assume

$$w(x_1,x_2,t)=w_0\ \exp[-k_1x_2]\exp[i(\omega t-kx_1)], \tag{31}$$

$$\psi(x_1,x_2,t)=\psi_0\ \exp[-k_2x_2]\exp[i(\omega t-kx_1)], \tag{32}$$

$$A_1(x_1,x_2,t)=A_{10}\ \exp[-k_1x_2]\exp[i(\omega t-kx_1)], \tag{33}$$

$$A_2(x_1,x_2,t)=A_{20}\ \exp[-k_1x_2]\exp[i(\omega t-kx_1)], \tag{34}$$

where $k^2-k_1^2=(\omega/c_a)^2$, $k^2-k_2^2=(\omega/c_\ell)^2$. The amplitude coefficients w_0, ψ_0, A_{10}, and A_{20} are not independent. Substituting w, A_1, and A_2 into the Lorentz gauge constraint (17), one may find the following relationships,

$$A_{10}=-\frac{\mu_0 e_{15}c_\ell \omega k}{(k_1^2-k^2)+\omega^2/c_\ell^{\,2}}w_0, \tag{35}$$

$$A_{20}=i\frac{\mu_0 e_{15}c_\ell \omega k_1}{(k_1^2-k^2)+\omega^2/c_\ell^{\,2}}w_0. \tag{36}$$

It should be noted that expressions (35) and (36) provide quantitative evaluation of the electromagneto-acoustic interaction caused by the piezoelectricity.

By substituting (31)–(34) into the boundary conditions (29)–(30) and by considering the relations (35) and (36), the boundary conditions (29)–(30) lead to the following homogeneous algebraic equations,

$$\begin{pmatrix} \left(\tilde{c}_{44}+\dfrac{\mu_0 e_{15}^2\omega}{(k_1^2-k^2)+\omega^2/c_\ell^{\,2}}\right)k_1 & e_{15}k_2 \\ \dfrac{e_{15}}{\epsilon_{11}^s}f & 1 \end{pmatrix}\begin{bmatrix} w_0 \\ \psi_0 \end{bmatrix}=0. \tag{37}$$

The nontrivial solution exists if and only if

$$\begin{vmatrix} \left(\tilde{c}_{44}+\dfrac{\mu_0 e_{15}^2\omega}{(k_1^2-k^2)+\omega^2/c_\ell^{\,2}}\right)k_1 & e_{15}k_2 \\ \dfrac{e_{15}}{\epsilon_{11}^s}f & 1 \end{vmatrix}=0, \tag{38}$$

which yields

$$k_1=\frac{e_{15}{}^2}{\epsilon_{11}^s\overline{c_{44}}}fk_2. \tag{39}$$

Let

$$\tilde{\beta}^2:=\frac{e_{15}{}^2}{\epsilon_{11}^s\overline{c_{44}}}f=\beta^2 f, \tag{40}$$

$$\tilde{f}:=\frac{c_\ell{}^2}{c_\ell{}^2-\tilde{\beta}^4 c_a{}^2}, \tag{41}$$

where the constant, $\beta:=e_{15}/\sqrt{\epsilon_{11}^s\overline{c_{44}}}$, is the piezoelectric coupling coefficient in the quasi-static approximation. To this end, a simple velocity equation for the electromagneto-acoustic surface wave is derived,

$$v_e=c_a\sqrt{\tilde{f}(1-\tilde{\beta}^4)}. \tag{42}$$

As $c_a/c_\ell\rightarrow 0$,

$$f=\frac{1}{1-c_a{}^2/c_\ell{}^2}\Rightarrow 1, \tag{43}$$

$$\tilde{\beta}^2=\beta^2 f\Rightarrow\beta^2, \tag{44}$$

$$\tilde{f}=\frac{1}{1-\tilde{\beta}^4 c_a{}^2/c_\ell{}^2}\Rightarrow 1; \tag{45}$$

the surface wave speed recovers the classic BG solution,

$$v_q=c_a\sqrt{1-\beta^4}. \tag{46}$$

Problem 2 (The free surface solution). In this case, the surface of the piezoelectric half space is a free surface, which is in contact with a vacuum half space on the top. The full set of boundary conditions at $x_2 = 0$ are as follows,[5]

$$\mathbf{n}\cdot\boldsymbol{\sigma}=0, \tag{47}$$

$$\mathbf{n}\cdot\mathbf{D}=\mathbf{n}\cdot\hat{\mathbf{D}}, \tag{48}$$

$$\mathbf{n}\times\mathbf{E}=\mathbf{n}\times\hat{\mathbf{E}}. \tag{49}$$

Note that the quantities with symbol ^ on the top denote the quantities of the electric field in the free vacuum space.

For this particular boundary configuration, the above boundary conditions can be put into explicit forms, i.e., at $x_2=0$,

$$\tilde{c}_{44}\frac{\partial w}{\partial x_2}+e_{15}\frac{\partial\psi}{\partial x_2}+\frac{e_{15}}{c_\ell}\frac{\partial A_2}{\partial t}=0, \tag{50}$$

$$e_{15}(1-f)\frac{\partial w}{\partial x_2}-\epsilon_{11}^s\frac{\partial\psi}{\partial x_2}-\frac{\epsilon_{11}^s}{c_\ell}\frac{\partial A_2}{\partial t}=-\epsilon_0\frac{\partial\hat{\phi}}{\partial x_2}-\frac{\epsilon_0}{c_0}\frac{\partial\hat{A}_2}{\partial t}, \tag{51}$$

$$\frac{\partial\psi}{\partial x_1}+\frac{e_{15}}{\epsilon_{11}^s}f\frac{\partial w}{\partial x_1}+\frac{1}{c_\ell}\frac{\partial A_1}{\partial t}=\frac{\partial\hat{\phi}}{\partial x_1}+\frac{1}{c_0}\frac{\partial\hat{A}_1}{\partial t}. \tag{52}$$

In addition, one may also note that the boundary condition for magnetic induction is always fulfilled, i.e.,

$$\mathbf{n}\cdot\mathbf{B}=\mathbf{n}\cdot\hat{\mathbf{B}}\equiv 0. \tag{53}$$

The Maxwell equations in the free vacuum half space take the form

$$\nabla^2\hat{\phi}-\frac{1}{c_0^2}\frac{\partial^2\hat{\phi}}{\partial t^2}=0, \tag{54}$$

$$\nabla^2\hat{A}_1-\frac{1}{c_0^2}\frac{\partial^2\hat{A}_1}{\partial t^2}=0, \tag{55}$$

$$\nabla^2\hat{A}_2-\frac{1}{c_0^2}\frac{\partial^2\hat{A}_2}{\partial t^2}=0, \tag{56}$$

where $c_0:=(\mu_0\epsilon_0)^{-1/2}$.

The associated Lorentz gauge in the vacuum space is

$$\frac{\partial \hat{A}_1}{\partial x_1}+\frac{\partial \hat{A}_2}{\partial x_2}+\frac{1}{c_0}\frac{\partial \hat{\phi}}{\partial t}=0. \tag{57}$$

Assume

$$\hat{\phi}=\hat{\phi}_0 \exp[k_3 x_2] \exp[i(\omega t-kx_1)], \tag{58}$$

$$\hat{A}_1=\hat{A}_{10} \exp[k_3 x_2] \exp[i(\omega t-kx_1)], \tag{59}$$

$$\hat{A}_2=\hat{A}_{20} \exp[k_3 x_2] \exp[i(\omega t-kx_1)], \tag{60}$$

where $k_3^2=k^2-\omega^2/c_0^2$.

To satisfy the Lorentz gauge constraint (57), the amplitude coefficients $\hat{\phi}_0$, $\hat{A}_{10}$, and $\hat{A}_{20}$ are related by

$$\hat{A}_{20}=\frac{i}{k_3}(k\hat{A}_{10}-(\omega/c_0)\hat{\phi}_0). \tag{61}$$

Substituting (58)–(60) into the boundary conditions (50)–(52) and considering (61), one may obtain the following linear homogeneous equations,

$$\overline{c_{44}}k_1\omega_0+e_{15}k_2\psi_0=0, \tag{62}$$

$$\epsilon_{11}^s k_2 k_3 \psi_0+\epsilon_0 k^2[\hat{\phi}_0-(v/c_0)\hat{A}_{10}]=0, \tag{63}$$

$$(e_{15}/\epsilon_{11}^s)\omega_0+\psi_0-[\hat{\phi}_0-(v/c_0)\hat{A}_{10}]=0. \tag{64}$$

Eliminate $[\hat{\phi}_0-(v/c_0)\hat{A}_{10}]$ from the above equations. It then yields

$$\begin{pmatrix} \overline{c_{44}}k_1 & e_{15}k_2 \\ \epsilon_{15}/\epsilon_{11}^s & 1+(\epsilon_{11}^s\epsilon_0)\left(\dfrac{k_2k_3}{k^2}\right)\end{pmatrix}\begin{bmatrix}\omega_0\\ \psi_0\end{bmatrix}=0. \tag{65}$$

Letting $v=\omega/k$ and equating the coefficient determinant to zero, one can find that the electromagneto-acoustic surface wave speed must obey the following velocity equation,

$$\overline{c_{44}}\epsilon_{11}^s\sqrt{1-(v_e/c_a)^2}\sqrt{1-(v_e/c_{\parallel})^2}\sqrt{1-(v_e/c_0)^2}$$
$$+\epsilon_0\overline{c_{44}}\sqrt{1-(v_e/c_a)^2}=\frac{\epsilon_0}{\epsilon_{11}^s}e_{15}^2\sqrt{1-(v_e/c_{\parallel})^2}. \tag{66}$$

If $v/c_{\parallel}\ll 1$, $v/c_0\ll 1$, it reduces to

$$\overline{c_{44}}(\epsilon_{11}^s+\epsilon_0)\sqrt{1-v^2/c_a^2}=\frac{\epsilon_0}{\epsilon_{11}^s}e_{15}^2. \tag{67}$$

Let

$$\beta_v^2:=\frac{e_{15}^2}{\overline{c_{44}}\epsilon_{11}^s}\frac{\epsilon_0}{(\epsilon_{11}^s+\epsilon_0)}. \tag{68}$$

Once again, we recover the classic result

$$v_q=c_a\sqrt{1-\beta_v^4}. \tag{69}$$

Remark 3.1. Assume $c_a<c_{\parallel}<c_0$. One can readily verify that the velocity equation (66) has no real root in the speed range $c_a\leqslant v\leqslant c_0$. Equation (66) may have complex roots within this speed range, or it may have a real root after $v>c_0$. Some of these possibilities may imply the existence of the electromagnetic surface wave on the interface between vacuum space and the piezoelectric medium. The quasi-static approximation usually fails to predict such possibility.

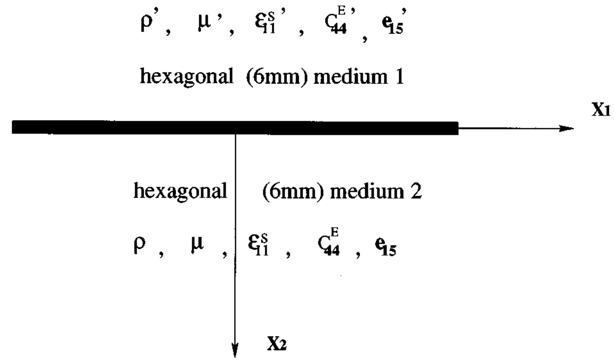

FIG. 2. An interface between two different piezoelectric media .

Problem 3 (the interface solution). This is an analogy of the well-known Maerfeld–Tournois surface wave that is also obtained under the quasi-static approximation.[6] The interface is located at $x_2=0$ between two different piezoelectric half spaces (see Fig. 2). In this case, the independent boundary conditions are as follows:

$$\mathbf{u}=\mathbf{u}', \tag{70}$$

$$\mathbf{n}\cdot\boldsymbol{\sigma}=\mathbf{n}\cdot\boldsymbol{\sigma}', \tag{71}$$

$$\mathbf{n}\cdot\mathbf{D}=\mathbf{n}\cdot\mathbf{D}', \tag{72}$$

$$\mathbf{n}\times\mathbf{E}=\mathbf{n}\times\mathbf{E}'. \tag{73}$$

One can verify that the magnetic boundary conditions,

$$\mathbf{n}\cdot\mathbf{B}=\mathbf{n}\cdot\mathbf{B}', \tag{74}$$

$$\mathbf{n}\times\mathbf{H}=\mathbf{n}\times\mathbf{H}', \tag{75}$$

are satisfied automatically.

The boundary conditions (70)–(73) can be written explicitly as

$$w=w', \tag{76}$$

$$\tilde{c}_{44}\frac{\partial w}{\partial x_2}+e_{15}\frac{\partial \psi}{\partial x_2}+\frac{e_{15}}{c_{\parallel}}\frac{\partial A_2}{\partial t}=\tilde{c}_{44}{}'\frac{\partial w'}{\partial x_2}+e_{15}{}'\frac{\partial \psi'}{\partial x_2}+\frac{e_{15}{}'}{c_{\parallel}{}'}\frac{\partial A_2{}'}{\partial t}, \tag{77}$$

$$e_{15}(1-f)\frac{\partial w}{\partial x_2}-\epsilon_{11}^s\frac{\partial \psi}{\partial x_2}-\frac{\epsilon_{11}^s}{c_{\parallel}}\frac{\partial A_2}{\partial t}=e_{15}{}'(1-f')\frac{\partial w'}{\partial x_2}-\epsilon_{11}^s{}'\frac{\partial \psi'}{\partial x_2}-\frac{\epsilon_{11}^s{}'}{c_{\parallel}{}'}\frac{\partial A_2{}'}{\partial t}, \tag{78}$$

$$\frac{\partial \psi}{\partial x_1}+\frac{e_{15}}{\epsilon_{11}^s}f\frac{\partial w}{\partial x_1}+\frac{1}{c_{\parallel}}\frac{\partial A_1}{\partial t}=\frac{\partial \psi'}{\partial x_1}+\frac{e_{15}{}'}{\epsilon_{11}^s{}'}f'\frac{\partial w'}{\partial x_1}+\frac{1}{c_{\parallel}{}'}\frac{\partial A_1{}'}{\partial t}. \tag{79}$$

For the lower half piezoelectric space, we still assume that

$$w=w_0\exp[-k_1x_2]\exp[i(\omega t-kx_1)], \tag{80}$$

$$\psi=\psi_0\exp[-k_2x_2]\exp[i(\omega t-kx_1)], \tag{81}$$

$$A_1 = A_{10}\exp[-k_1x_2]\exp[i(\omega t - kx_1)], \tag{82}$$

$$A_2 = A_{20}\exp[-k_1x_2]\exp[i(\omega t - kx_1)], \tag{83}$$

with $k^2 - k_1^2 = (\omega/c_a)^2$, $k^2 - k_2^2 = (\omega/c_{\ell})^2$; and consequently,

$$A_{10} = \frac{-\mu_0 e_{15} c_{\ell}\omega k}{[(k_1^2 - k^2) + \omega^2/c_{\ell}^2]}w_0, \tag{84}$$

$$A_{20} = \frac{i\mu_0 e_{15} c_{\ell}\omega k_1}{[(k_1^2 - k^2) + \omega^2/c_{\ell}^2]}w_0. \tag{85}$$

For the upper half piezoelectric space, we assume that

$$w' = w_0'\exp[k_1'x_2]\exp[i(\omega t - kx_1)], \tag{86}$$

$$\psi' = \psi_0'\exp[k_2']\exp[i(\omega t - kx_1)], \tag{87}$$

$$A_1' = A_{10}'\exp[k_1'x_2]\exp[i(\omega t - kx_1)], \tag{88}$$

$$A_2' = A_{20}'\exp[k_1'x_2]\exp[i(\omega t - kx_1)]; \tag{89}$$

with $k^2 - (k_1')^2 = (\omega/c_a')^2$, $k^2 - (k_2')^2 = (\omega/c_{\ell}')^2$; and consequently,

$$A_{10}' = \frac{-\mu_0 e_{15} c_{\ell}\omega k}{[((k_1')^2 - k^2) + \omega^2/c_{\ell}^2]}w_0', \tag{90}$$

$$A_{20}' = \frac{-i\mu_0 e_{15} c_{\ell}\omega k_1'}{[((k_1')^2 - k^2) + \omega^2/c_{\ell}^2]}w_0'. \tag{91}$$

Inserting the assumed solutions (80)–(83) and (86)–(89) into the boundary condition set (76)–(79) and taking equations (84) and (85) and (90) and (91) into consideration, one may find the following existence condition for the interfacial electromagneto-acoustic wave:

$$\begin{vmatrix} 1 & 0 & -1 & 0 \\ k_1\overline{c_{44}} & e_{15}k_2 & k_1'\overline{c_{44}'} & e_{15}'k_2' \\ 0 & \epsilon_{11}^s k_2 & 0 & \epsilon_{11}^{s\prime}k_2' \\ (e_{15}/\epsilon_{11}^s)k & k & -(e_{15}'/\epsilon_{11}^{s\prime})k & -k \end{vmatrix} = 0. \tag{92}$$

Subsequently, it leads the following wave velocity equation,

$$\begin{aligned}&\overline{c_{44}}\epsilon_{11}^s\sqrt{1-(v_e/c_a)^2}\sqrt{1-(v_e/c_{\ell})^2}+\overline{c_{44}}\epsilon_{11}^{s\prime}\sqrt{1-(v_e/c_a)^2}\\ &\times\sqrt{1-(v_e/c_{\ell}')^2}+\overline{c_{44}}'\epsilon_{11}^s\sqrt{1-(v_e/c_a')^2}\\ &\times\sqrt{1-(v_e/c_{\ell})^2}+\overline{c_{44}}'\epsilon_{11}^{s\prime}\sqrt{1-(v_e/c_a')^2}\\ &\times\sqrt{1-(v_e/c_{\ell}')^2}=\Delta\sqrt{1-(v_e/c_{\ell})^2}\sqrt{1-(v_e/c_{\ell}')^2},\end{aligned} \tag{93}$$

where

$$\Delta := \left[\left(\frac{e_{15}}{\epsilon_{11}^s}\right) - \left(\frac{e_{15}'}{\epsilon_{11}^{s\prime}}\right)\right]^2 \epsilon_{11}^s\epsilon_{11}^{s\prime}. \tag{94}$$

When $v/c_{\ell} << 1$, $v/c_{\ell}' \ll 1$, the above velocity equation reduces to the Maerfeld-Tournois wave speed equation,[6]

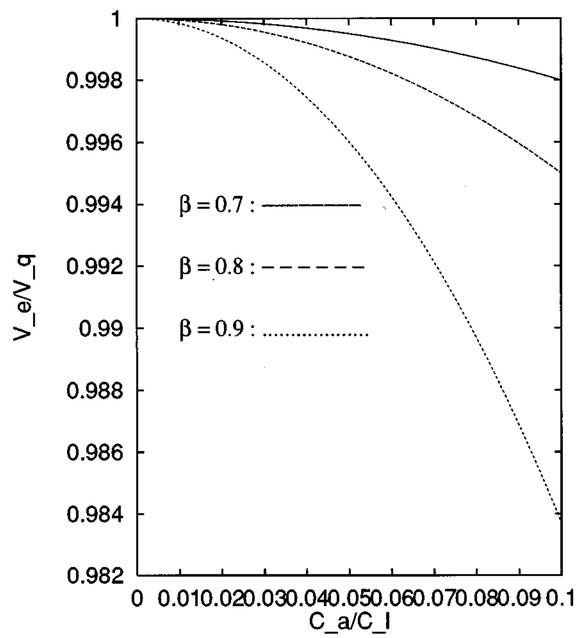

FIG. 3. The ratio v_e/v_q versus c_a/c_{ℓ} for the grounded surface solution .

$$\begin{aligned}&\overline{c_{44}}\sqrt{1-(v_q^2/c_a)^2}+\overline{c_{44}}'\sqrt{1-(v_q/c_a')^2}\\ &=\left[\left(\frac{e_{15}}{\epsilon_{11}^s}\right)-\left(\frac{e_{15}'}{\epsilon_{11}^{s\prime}}\right)\right]^2\left(\frac{\epsilon_{11}^s\epsilon_{11}^{s\prime}}{\epsilon_{11}^s+\epsilon_{11}^{s\prime}}\right).\end{aligned} \tag{95}$$

Remark 3.2. Assume $c_a \leqslant c_a' \leqslant c_{\ell} \leqslant c_{\ell}'$. The velocity equation (93) can only have complex roots in the speed range $c_a' \leqslant v \leqslant c_{\ell}$. After $v > c_{\ell}$, it may have real roots. Once again, we cannot rule out the possibility that there is a piezoelectrically-induced electromagnetic surface wave propagating along the interface.

IV. CONCLUSIONS

As shown above, the exact solutions for the fully-coupled SH electromagneto-acoustic surface wave can be obtained in simple, closed forms. As expected, these solutions take the classic results, such as BG wave or Maerfeld–Tournois wave, as their limits, when the terms v/c_{ℓ} and v/c_0 are negligible.

The new results obtained here reveal the acousto-optic interaction on the surface of piezoelectric materials, and implicitly indicate a possible existence of the piezoelectrically-induced electromagnetic surface wave.

As far as the acoustic wave is concerned, the quasi-static assumption does offer an excellent approximation for the wave speed. This can be observed from Fig. 3. In Fig. 3, a comparison between the current result and the classic result is made for *Problem 1*, the grounded surface solution. Since in reality the ratio c_a/c_{ℓ} is within the range of $0 \sim 0.0001$, the optical effect on acoustic wave speed is minor indeed.

However, the quasi-static approximation may not always make life easy; it can cause some additional difficulties. Such problems arise when one studies wave propagation or wave scattering problems in a piezoelectric material containing defects.[7,8] An obvious reason for this is that the governing equations obtained under quasi-static approximation are mixed types of partial differential equations; namely, one is hyperbolic, the other is elliptic, because the speed of light is assumed to be infinite in the quasi-static approximation. By using the approach proposed in this paper, one may obtain equally simple decoupled governing equations, and they are both hyperbolic type of partial differential equations.

It should be reminded that all the surface waves studied here are nondispersive. There may exist dispersive electromagneto-acoustic surface wave mode in some piezoelectric solids with particular geometric shapes, such as an infinite piezoelectric plate. Under those circumstances, there may be a nontrivial discrepancy for the high frequency surface wave solutions between the approach proposed in this paper and the quasi-static approximation. This issue is subjected the further study.

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[1] J. L. Bleustein, Appl. Phys. Lett. **13**, 412 (1968).
[2] Y. V. Gulyaev, Sov. Phys. JETP **9**, 37 (1969).
[3] B. A. Auld, *Acoustic Field and Wave in Solids* (Wiley, New York, 1990), Vol. 1.
[4] B. A. Auld, *Acoustic Field and Wave in Solids* (Wiley, New York, 1990), Vol. 2.
[5] J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975).
[6] C. Maerfeld and P. Tournois, Appl. Phys. Lett. **19**, 117 (1971).
[7] S. Li and P. A. Mataga, J. Mech. Phys. Solids (to be published).
[8] S. Li and P. A. Mataga, J. Mech. Phys. Solids (to be published).