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On diffraction in a piezoelectric medium by a half-plane: The Sommerfeld problem

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Abstract. This paper is concerned with the diffraction problem in a transversely isotropic piezoelectric medium by a half-plane. The half-plane obstacle considered here is a semi-infinite slit, or a crack; both its surfaces are traction free and electric absorbent screens. In a generalized sense, we are dealing with the Sommerfeld problem in a piezoelectric medium.

The coupled diffraction fields between acoustic wave and electric wave are excited by both incident acoustic wave as well as incident electric wave; and the sound soft and electric "blackness" conditions on the screens are characterized by a system of simultaneous Wiener-Hopf equations. Closed form solutions are sought by employing special techniques. Some interesting results have been obtained, such as mode conversions between acoustic wave and electric wave, novel diffraction patterns in the scattering fields, and the effect of electroacoustic head wave, as well as of surface wave—Bleustein-Gulyaev wave.

Unlike the classical Sommerfeld problem, in which the only concern is the scattering field of electric wave, the strength of material, e.g. material toughness, is another concern here. From this perspective, relevant dynamic field intensity factors at the crack tip are derived explicitly.

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1. Introduction

The study of wave scattering problems in piezoelectric media is of particular importance, because many piezoelectric devices are exclusively made as wave guides, which can either enhance acoustic wave, or transfer acoustic energy to electric energy, and vice versa for practical purposes. In fact, wave propagation in the piezoelectric medium is a unique embodiment of acoustic wave and electromagnetic wave, both of which are the paradigms of linear hyperbolic systems of partial differential equations, which attributes special significance to such study. Unfortunately, if not surprisingly, it appears to this author that there is a lack of fundamental understanding on the subject. This work attempts to provide a systematic analysis on a half-plane scattering problem in a transversely isotropic piezoelectric medium, which is generated by both shear-horizontal (SH) acoustic wave and transverse electric (TE) wave.

There is a dilemma in studying scattering problems in piezoelectric media, even if one is only interested in electroacoustic wave.† In general, the fully coupled Christoffel-Maxwell or Euler-Maxwell equations are hardly tractable. In order to simplify the problem, quasi-static approximation is widely adopted. The setback of quasi-static approximation is that it leads to the loss of hyperbolicity of the simplified system, and subsequently prevent any meaningful analysis on transient problems in piezoelectric medium. Almost all of the previous attempts on the subject were made within the realm of quasi-static approximation, consequently, the results obtained are numeric in nature (e.g. Auld [1973b], Parton & Kudryatvsev [1988], and Shindo et al. [1990]). To improve the situation, Li [1998] proposed a so-called "*quasi-hyperbolic approximation*" for a class of transversely isotropic piezoelectric media, and the purpose of this "quasi-hyperbolic approximation" is to preserve the hyperbolicity of the simplified system, and at the same time the simplified system can still enjoy the simplicity that the "quasi-static" approximation provided before.

Consider a traction free and perfectly conducting crack (with its both surfaces as absorbent screens), which is located at the positive part of x_1 axis, namely,

$$\left\{\begin{aligned} \sigma_{23}(x_1,0,t) &= \sigma_{23}^{(s)}(x_1,0,t)+\sigma_{23}^{(i)}(x_1,0,t)=0\ , \quad (a)\\ \phi(x_1,0,t) &= \phi^{(s)}(x_1,0,t)+\phi^{(i)}(x_1,0,t)=0\ , \quad (b)\end{aligned}\right. \qquad 0<x_1<+\infty \tag{1.1}$$

The condition (1.1(b)) (absorbent screen) renders it as a *Sommerfeld problem* (Sommerfeld [1896], [1901], and [1949]) in a generalized sense. From the point of view of mathematical physics, this is a mixed Dirichlet-Neumann, or Robin problem; the interplay between mechanical field and electric field along the boundary generates both symmetric and anti-symmetric scattering fields. Subsequently, there may exist a nonzero bulk scattering displacement field as well as non-zero electrical potential field at the crack tip, which is fundamentally different from the conventional diffraction problems by cracks (there are some good examples in Sih [1977]).

2. Problem statement

Based on the "quasi-hyperbolic approximation" (see Li [1998]), for hexagonal symmetry piezoelectric materials (e.g. $6mm$ class), the relevant electromechanical

† In this paper, the term *electro-acoustic wave* is reserved for electric potential disturbance travelling at sound speed; whereas the term *electroacoustic wave* is referred to as both acoustic wave as well as electro-acoustic wave.

coupling on transverse plane is between anti-plane displacement and in-plane electric field, i.e.

$$\mathbf{u} = \big(0, 0, w(x_1, x_2, t)\big) \tag{2.1}$$

$$\mathbf{E} = \big(-\frac{\partial \phi}{\partial x_1}, -\frac{\partial \phi}{\partial x_2}, 0\big) \tag{2.2}$$

which can then be translated into the coupling between SH acoustic wave and TE electric wave. Introduce a pseudo-electric potential function

$$\psi = \phi - \frac{e_{15}}{\epsilon_{11}^s} C_f w \tag{2.3}$$

where $C_f := {c_\ell}^2/({c_\ell}^2 - c_s^2)$, $c_\ell := 1/\sqrt{\epsilon_{11}^s \mu_0}$, $c_s := \sqrt{\bar{c}_{44}/\rho}$, and $\bar{c}_{44} := c_{44}^E + {e_{15}}^2/\epsilon_{11}^s$.

Following Li [1996], we then have a system of decoupled wave equations,

$$\Big(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} - \frac{1}{c_s^2}\frac{\partial^2}{\partial t^2}\Big) w = 0 \tag{2.4}$$

$$\Big(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} - \frac{1}{{c_\ell}^2}\frac{\partial^2}{\partial t^2}\Big) \psi = 0 \ . \tag{2.5}$$

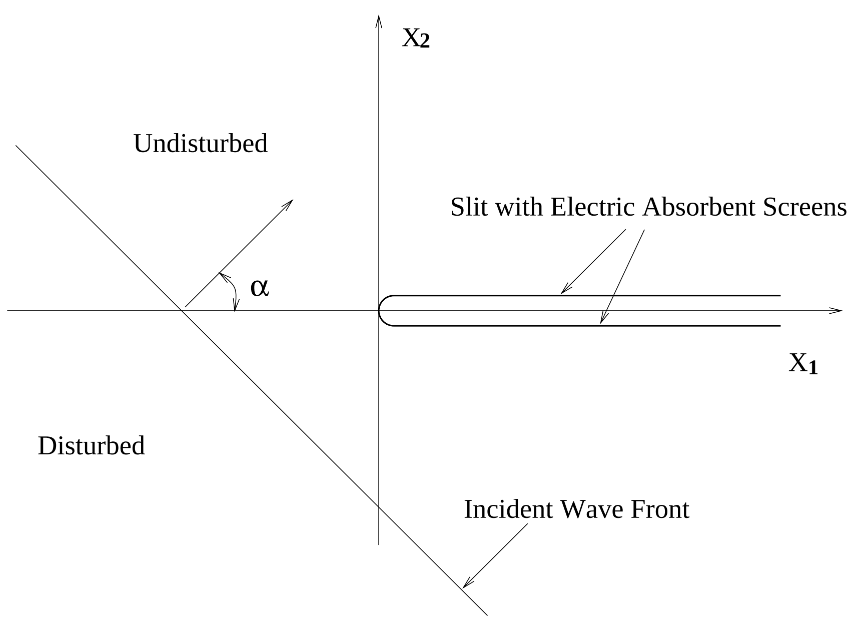

Figure 1.
Schematic illustration of incident wave approaching a semi-infinite slit

Under the quasi-hyperbolic approximation, the relevant constitutive equations are

$$\sigma_{13} = \tilde{c}_{44} \frac{\partial w}{\partial x_1} + e_{15} \frac{\partial \psi}{\partial x_1} \tag{2.6}$$

$$\sigma_{23} = \tilde{c}_{44} \frac{\partial w}{\partial x_2} + e_{15} \frac{\partial \psi}{\partial x_2} \tag{2.7}$$

$$D_1 = e_{15}(1 - C_f) \frac{\partial w}{\partial x_1} - \epsilon^s_{11} \frac{\partial \psi}{\partial x_1} \tag{2.8}$$

$$D_2 = e_{15}(1 - C_f) \frac{\partial w}{\partial x_2} - \epsilon^s_{11} \frac{\partial \psi}{\partial x_2} \tag{2.9}$$

where $\tilde{c}_{44} := \bar{c}_{44}\big[1 - (1 - C_f)({e_{15}}^2/\bar{c}_{44}\epsilon^s_{11})\big]$.

The total solution of the scattering problem consists of two parts

$$w = w^{(s)} + w^{(i)} \tag{2.10}$$

$$\psi = \psi^{(s)} + \psi^{(i)} \tag{2.11}$$

The superscript "(s)" indicates the scattering field, and the superscript "(i)" indicates the incident field. The incident acoustic wave as well as the incident electric wave are assumed to be in the form of plane wave,

$$w^{(i)}(x_1, x_2, t) = G_w\big(t - s_s[\cos(\alpha_w)x_1 + \sin(\alpha_w)x_2]\big) \tag{2.12}$$

$$\psi^{(i)}(x_1, x_2, t) = G_\psi\big(t - s_\ell[\cos(\alpha_\psi)x_1 + \sin(\alpha_\psi)x_2]\big) \tag{2.13}$$

where $G_w,\ G_\psi$ are given functions, and $s_s := 1/c_s,\ s_\ell := 1/c_\ell$ are the slownesses.

In Eq.(2.12) and (2.13), $0 \le \alpha_w, \alpha_\psi \le \pi/2$ are angles of incident waves. In this paper, we assume that $\alpha_w = \alpha_\psi = \alpha$, though, in principle, the angle of the acoustic incident wave, α_w, can be different from the angle of electric incident wave, α_ψ. Similarly, the shape function of the acoustic incident wave, G_w, is also different from the shape function of electric incident wave, G_ψ, in general; nonetheless for simplicity, we assume

$$G_w(t) := w_0 G(t)\ ,\ G_\psi(t) := \psi_0 G(t)$$

$$G(t) := H(t) \int_0^t g(\tau) d\tau \tag{2.14}$$

where $g(\cdot)$ is a given function and w_0, ψ_0 are the amplitudes of acoustic incident wave and electric incident wave respectively.

The incident disturbance of electric potential is then the combination of incident acoustic wave and electric wave,

$$\phi^{(i)}(x_1, x_2, t) = \psi^{(i)}(x_1, x_2, t) + \frac{e_{15}}{\epsilon^s_{11}} C_f w^{(i)}(x_1, x_2, t) \tag{2.15}$$

The boundary conditions on the crack surfaces, or screens are

$$\begin{cases} \tilde{c}_{44}\dfrac{\partial w^{(s)}}{\partial x_2} + e_{15}\dfrac{\partial \psi^{(s)}}{\partial x_2} = -\tilde{c}_{44}\dfrac{\partial w^{(i)}}{\partial x_2} - e_{15}\dfrac{\partial \psi^{(i)}}{\partial x_2} \\ \psi^{(s)} + \dfrac{e_{15}}{\epsilon_{11}^s}C_f w^{(s)} = -\psi^{(i)} - \dfrac{e_{15}}{\epsilon_{11}^s}C_f w^{(i)} \end{cases} \quad 0 \le x_1 < \infty \ , \ x_2 = 0 \tag{2.16}$$

For scattering fields, the following initial conditions and radiation conditions are imposed as

$$w^{(s)}(x_1, x_2, t) = \dot{w}^{(s)}(x_1, x_2, t) = 0 \ , \ t < 0 \tag{2.17}$$

$$\psi^{(s)}(x_1, x_2, t) = \dot{\psi}^{(s)}(x_1, x_2, t) = 0 \ , \ t < 0 \tag{2.18}$$

and

$$\lim_{r\to\infty}\Big[w^{(s)}, \psi^{(s)}, \dot{w}^{(s)}, \dot{\psi}^{(s)}, etc.\Big] = 0 \ , \ t > 0 \tag{2.19}$$

The edge condition will be considered wherever is needed.

3. Solutions of Wiener-Hopf equations

The diffraction problem by a half-plane is amenable to the Wiener-Hopf technique, which has been a powerful analytical apparatus that is responsible for solving a number of benchmark scattering problems in both acoustics as well as electromagnetics. Even though this function-theoretic method is well established (e.g. Noble [1958], Jones [1964], and Mittra & Lee [1971]), in general there is still no systematic procedure to follow in dealing with simultaneous Wiener-Hopf equations, which is usually the case in coupling problems, for instance, this particular problem. At following, a novel procedure is carried out to solve the related simultaneous Wiener-Hopf equations.

(a) Transform solutions

Applying the following double Laplace transforms

$$\begin{cases} f^*(x, p) = \displaystyle\int_0^\infty f(x, t)\exp(-pt)dt \\ f(x, t) = \dfrac{1}{2\pi i}\displaystyle\int_{p_0 - i\infty}^{p_0 + i\infty} f^*(x, p)\exp(pt)dp \end{cases} \tag{3.1}$$

$$\begin{cases} \hat{f}^*(\zeta, p) = \displaystyle\int_{-\infty}^\infty f^*(x, p)\exp(-p\zeta x)dx \\ f^*(x, p) = \dfrac{p}{2\pi i}\displaystyle\int_{\zeta_0 - i\infty}^{\zeta_0 + i\infty} \hat{f}^*(\zeta, p)\exp(p\zeta x)d\zeta \end{cases} \tag{3.2}$$

to wave equations (2.4), (2.5) yields

$$\Big(\frac{d^2}{dx_2^2} - p^2 a^2(\zeta)\Big)\hat{w}^*(\zeta, x_2, p) = 0 \tag{3.3}$$

$$\Big(\frac{d^2}{dx_2^2} - p^2 e^2(\zeta)\Big)\hat{\psi}^*(\zeta, x_2, p) = 0 \tag{3.4}$$

where $a(\zeta) = \sqrt{s_s^2 - \zeta^2}$ and $e(\zeta) = \sqrt{s_\ell^2 - \zeta^2}$.

Consequently, there exist constants $\varepsilon_+, \varepsilon_-$ such that $-s_\ell < \varepsilon_+ < \varepsilon_- < s_\ell$, and they determine a pair of overlapped half planes P_+ and P_-:

$$P_+ := \{\zeta \in \mathbb{C} \,\Big|\, Re(\zeta) > \varepsilon_+\} \tag{3.5}$$

$$P_- := \{\zeta \in \mathbb{C} \,\Big|\, Re(\zeta) < \varepsilon_-\} \tag{3.6}$$

Because of the nature of the mixed boundary value problem, the scattering fields governed by (3.3) and (3.4) are neither symmetric nor anti-symmetric. As the result, general form of solutions has to be sought,

$$\begin{cases} \hat{w}^*(\zeta, x_2, p) = \Big(A_{an}(\zeta, p)\,\mathrm{sgn}(x_2) + A_{sy}(\zeta, p)\Big)\exp\Big(-pa(\zeta)|x_2|\Big) \\ \hat{\psi}^*(\zeta, x_2, p) = \Big(B_{an}(\zeta, p)\,\mathrm{sgn}(x_2) + B_{sy}(\zeta, p)\Big)\exp\Big(-pe(\zeta)|x_2|\Big) \end{cases}$$

where $A_{an}(\zeta, p), B_{an}(\zeta, p)$ represent the anti-symmetry part of the solutions and $A_{sy}(\zeta, p), B_{sy}(\zeta, p)$ are the symmetry part of the solutions.

The general properties of anti-symmetry solution and symmetry solution have been well documented in the literature (e.g. Noble [1958], Jones [1952]). For easy reference, some useful properties of these functions are listed. Define

$$U_+(\zeta, x_2, p) := \int_0^\infty \int_0^\infty u(x_1, x_2, t)\exp\big(-p[t + \zeta x_1]\big)dtdx_1$$

$$U_-(\zeta, x_2, p) := \int_{-\infty}^0 \int_0^\infty u(x_1, x_2, t)\exp\big(-p[t + \zeta x_1]\big)dtdx_1$$

By the definition, the anti-symmetry solutions are

$$A_{an}(\zeta, p) = W_{an+}(\zeta, +0, p) + W_{an-}(\zeta, +0, p) \tag{3.7}$$

$$B_{an}(\zeta, p) = \Psi_{an+}(\zeta, +0, p) + \Psi_{an-}(\zeta, +0, p) \tag{3.8}$$

Then it can be deduced that

$$W_{an-}(\zeta, +0, p) = W_{an-}(\zeta, -0, p) = 0 \tag{3.9}$$

$$\Psi_{an-}(\zeta, +0, p) = \Psi_{an-}(\zeta, -0, p) = 0 \tag{3.10}$$

and

$$A_{an}(\zeta,p) = W_{an+}(\zeta,+0,p) = -W_{an+}(\zeta,-0,p) \tag{3.11}$$

$$B_{sn}(\zeta,p) = \Psi_{sn+}(\zeta,+0,p) = -\Psi_{an+}(\zeta,-0,p) \tag{3.12}$$

In parallel, the symmetry solutions have the form

$$A_{sy}(\zeta,p) = W_{sy+}(\zeta,+0,p) + W_{sy-}(\zeta,+0,p) \tag{3.13}$$

$$B_{sy}(\zeta,p) = \Psi_{sy+}(\zeta,+0,p) + \Psi_{sy-}(\zeta,+0,p) \tag{3.14}$$

and

$$a(\zeta)A_{sy}(\zeta,p) \in \mathcal{O}(P_+) \tag{3.15}$$

$$e(\zeta)B_{sy}(\zeta,p) \in \mathcal{O}(P_+) \tag{3.16}$$

where $\mathcal{O}(P_+)$ is defined as the set of sectionally analytic functions in the half-plane P_+,

$$\mathcal{O}(P_+) := \left\{ f(\zeta) \;\middle|\; f(\zeta) \textit{ is holomorphic }, \ \forall\, \zeta \in P_+ \right\} \tag{3.17}$$

Because of the difference between acoustic excitation and electric excitation, it is convenient to treat them separately. At following, we use the superscript "(sa)" denoting scattering acoustic field, and "(se)" denoting scattering electric field; for example

$$A^{(sa)}_{an}(\zeta,p) := W^{(sa)}_{an+}(\zeta,p) + W^{(sa)}_{an-}(\zeta,p) \tag{3.18}$$

and

$$B^{(se)}_{sy}(\zeta,p) := \Psi^{(se)}_{sy+}(\zeta,p) + \Psi^{(se)}_{sy-}(\zeta,p) \tag{3.19}$$

where simplified notations are used, i.e. $W^{(sa)}_{an\pm}(\zeta,p) := W^{(sa)}_{an\pm}(\zeta,+0,p)$ and $\Psi^{(se)}_{sy\pm}(\zeta,p) := \Psi^{(se)}_{sy\pm}(\zeta,+0,p)$. The same convention will be followed throughout the rest of the paper without further specification.

Solutions for acoustic incident wave

First, we consider the acoustic incident wave only, namely, $w_0 \neq 0$; $\psi_0 = 0$, though it should be reminded that in piezoelectric media acoustic wave is always accompanied by the disturbance of electric potential that travels at the same sound speed. In this case, the Wiener-Hopf equations derived from screen boundary conditions are

$$-\tilde{c}_{44}a(\zeta)\Big[A^{(sa)}_{an}(\zeta,p) + A^{(sa)}_{sy}(\zeta,p)\Big] - e_{15}e(\zeta)\Big[B^{(sa)}_{an}(\zeta,p) + B^{(sa)}_{sy}(\zeta,p)\Big] = \frac{\Sigma^{(sa)}_{-}(\zeta,p)}{p} + \Big[\frac{g^*(p)}{p}\Big]\frac{\tilde{c}_{44}s_s \sin\alpha w_0}{\zeta + s_s\cos\alpha} \tag{3.20}$$

$$\frac{e_{15}}{\epsilon^s_{11}}C_f\Big[W^{(sa)}_{an+}(\zeta,p) + W^{(sa)}_{sy+}(\zeta,p)\Big] + \Big[\Psi^{(sa)}_{an+}(\zeta,p) + \Psi^{(sa)}_{sy+}(\zeta,p)\Big] = -\Big[\frac{g^*(p)}{p}\Big]\Big(\frac{e_{15}}{\epsilon^s_{11}}C_f\Big)\frac{w_0}{\zeta + s_s\cos\alpha}\ , \tag{3.21}$$

where

$$\Sigma_-^{(sa)}(\zeta,p) := \int_{-\infty}^{0}\int_0^{\infty} \sigma_{23}^{(sa)}(x_1,+0,t)\exp\bigl(-p[\zeta x_1+t]\bigr)dtdx_1$$

The above Wiener-Hopf equations can be further split into two simultaneous dual Wiener-Hopf equations

$$\begin{cases} -\tilde{c}_{44}a(\zeta)A_{an}^{(sa)}(\zeta,p) - e_{15}e(\zeta)B_{an}^{(sa)}(\zeta,p) = \dfrac{\Sigma_-^{(sa)}(\zeta,p)}{p} + \Bigl[\dfrac{g^*(p)}{p}\Bigr]\dfrac{\bar{c}_{44}s_s\sin\alpha w_0}{\zeta+s_s\cos\alpha} \\ \frac{e_{15}}{\epsilon_{11}^s}C_f W_{an+}^{(sa)}(\zeta,p) + \Psi_{an+}^{(sa)}(\zeta,p) = 0 \end{cases} \tag{3.22}$$

and

$$\begin{cases} -\tilde{c}_{44}a(\zeta)A_{sy}^{(sa)}(\zeta,p) - e_{15}e(\zeta)B_{sy}^{(sa)}(\zeta,p) = 0 \\ \frac{e_{15}}{\epsilon_{11}^s}C_f W_{sy+}^{(sa)}(\zeta,p) + \Psi_{sy+}^{(sa)}(\zeta,p) = -\Bigl[\frac{g^*(p)}{p}\Bigr]\Bigl(\frac{e_{15}}{\epsilon_{11}^s}C_f\Bigr)\frac{w_0}{\zeta+s_s\cos\alpha} \end{cases} \tag{3.23}$$

where Equations (3.22) determine the anti-symmetry solution, whereas Equations (3.23) determine the symmetry solution.

Anti-symmetry solution

For anti-symmetry solutions,

$$W_{an-}^{(sa)}(\zeta,p) = \Psi_{an-}^{(sa)}(\zeta,p) = 0\ , \tag{3.24}$$

hence

$$A_{an}^{(sa)}(\zeta,p) = W_{an+}^{(sa)}(\zeta,p)\ ,\ \ B_{an}^{(sa)}(\zeta,p) = \Psi_{an+}^{(sa)}(\zeta,p)\ .$$

Consequently from (3.22), we have

$$\Psi_{an+}^{(sa)}(\zeta,p) = -\Bigl(\frac{e_{15}}{\epsilon_{11}^s}C_f\Bigr)W_{an+}^{(sa)}(\zeta,p) \tag{3.25}$$

$$-\bar{c}_{44}\Bigl(a(\zeta)-k_e^2e(\zeta)\Bigr)W_{an+}^{(sa)}(\zeta,p) = \frac{\Sigma_-^{(sa)}(\zeta,p)}{p} + \Bigl[\frac{g^*(p)}{p}\Bigr]\frac{\bar{c}_{44}s_s\sin\alpha w_0}{\zeta+s_s\cos\alpha} \tag{3.26}$$

where $k_e^2 := \dfrac{{e_{15}}^2}{\epsilon_{11}^s\tilde{c}_{44}}C_f$. Note that there is a slight difference between this definition of k_e and the traditional definition under the quasi-static approximation (e.g. Maugin [1983] page 393). Define the Bleustein-Gulyaev wave function

$$BG(\zeta) := a(\zeta) - k_e^2e(\zeta)\ . \tag{3.27}$$

A product decomposition of Bleustein-Gulyaev function is given in Li & Mataga [1996a] (also see Li [1998]),

$$BG(\zeta) = (1-k_e^2)\frac{(s_{bge}+\zeta)(s_{bge}-\zeta)}{\sqrt{(s_s+\zeta)(s_s-\zeta)}}\mathcal{S}_+(\zeta)\mathcal{S}_-(\zeta) \tag{3.28}$$

where

$$\mathcal{S}_\pm(\zeta) = \exp\left\{\frac{1}{\pi}\int_{s_\ell}^{s_s}\arctan\Big[\frac{k_e^2\sqrt{(\eta-s_\ell)(\eta+s_\ell)}}{\sqrt{(s_s-\eta)(s_s+\eta)}}\Big]\frac{d\eta}{\eta\pm\zeta}\right\} \tag{3.29}$$

Remark 3.1. The Bleustein-Gulyaev slowness is defined as

$$s_{bge} := \sqrt{\frac{s_s^2 - k_e^4 s_\ell^2}{1-k_e^4}} \tag{3.30}$$

Let $c_\ell \to \infty$, then $C_f \to 1$, $\tilde{c}_{44} \to \bar{c}_{44}$ and

$$c_{bge} := 1/s_{bge} \to c_s\sqrt{1-k_e^4}\ , \text{ and } k_e^2 \to \frac{{e_{15}}^2}{\epsilon_{11}^s\bar{c}_{44}}$$

which recovers the original definitions of Bleustein-Gulyaev wave speed and the electro-mechanical coupling coefficient, k_e, (Bleustein [1968], Gulyaev [1969]). □

The final decomposition then becomes

$$-(1-k_e^2)\frac{(s_{bge}+\zeta)}{\sqrt{(s_s+\zeta)}}\mathcal{S}_+(\zeta)W_{an+}^{(sa)}(\zeta,p) = \frac{\Sigma_-^{(sa)}(\zeta,p)}{p}\cdot\frac{M_-(\zeta)}{\tilde{c}_{44}} + \Big[\frac{g^*(p)}{p}\Big]\frac{M_-(-s_s\cos\alpha)}{(\zeta+s_s\cos\alpha)} + \Big[\frac{g^*(p)}{p}\Big]\frac{\big[M_-(\zeta)-M_-(-s_s\cos\alpha)\big]}{(\zeta+s_s\cos\alpha)} \tag{3.31}$$

where $M_-(\zeta) := \sqrt{(s_s-\zeta)}\mathcal{D}_-(\zeta)/(s_{bge}-\zeta)$ and $\mathcal{D}_\pm(\zeta) := 1/\mathcal{S}_\pm(\zeta)$.

The anti-symmetry solutions can be readily derived as

$$A_{an}^{(sa)}(\zeta,p) = -\Big[\frac{g^*(p)}{p}\Big]\Big(\frac{s_s\sin\alpha w_0}{1-k_e^2}\Big)\frac{M_-(-\zeta_{s\alpha})\sqrt{s_s+\zeta}}{(s_{bge}+\zeta)}\frac{\mathcal{D}_+(\zeta)}{(\zeta+\zeta_{s\alpha})}\ , \tag{a}$$

$$B_{an}^{(sa)}(\zeta,p) = \Big[\frac{g^*(p)}{p}\Big]\Big(\frac{e_{15}}{\epsilon_{11}^s}C_f\Big)\Big(\frac{s_s\sin\alpha w_0}{1-k_e^2}\Big)\frac{M_-(-\zeta_{s\alpha})\sqrt{s_s+\zeta}\mathcal{D}_+(\zeta)}{(s_{bge}+\zeta)(\zeta+\zeta_{s\alpha})}\ , \tag{b}$$

$$\Sigma_-^{(sa)}(\zeta,p) = \bar{c}_{44}g^*(p)\Big(\frac{s_s\sin\alpha w_0}{\zeta+\zeta_{s\alpha}}\Big)\Big[\frac{M_-(-\zeta_{s\alpha})}{M_-(\zeta)}-1\Big]\ , \tag{c}$$

(3.32)

where $\zeta_{s\alpha} := s_s \cos\alpha$.

Remark 3.2. Let piezoelectric coefficient $e_{15} \to 0$. From (3.32(a)) and (3.32(b)), one may find that

$$A_{an}^{(sa)} \to -\Big[\frac{g^*(p)}{p}\Big]\Big[\frac{s_s \sin\alpha w_0}{\sqrt{s_s(1+\cos\alpha)}}\Big]\frac{1}{(\zeta + s_s\cos\alpha)\sqrt{s_s+\zeta}} \tag{3.33}$$

$$B_{an}^{(sa)} \to 0 \ . \tag{3.34}$$

Eq. (3.33) is the exact same result for the diffraction of purely acoustic SH wave by a slit (See Achenbach [1973] page 376). □

Symmetry solutions

In contrast to anti-symmetry solution, to find the symmetry solutions, $A_{sy}^{(sa)}(\zeta,p)$ and $B_{sy}^{(sa)}(\zeta,p)$, is not straightforward, because in this case both $W_{sy-}^{(sa)}(\zeta,p) \neq 0$ and $\Psi_{sy-}^{(sa)}(\zeta,p) \neq 0$. What follows is a "semi-inverse" type of procedure to construct symmetry solutions, meaning that the key derivation step is relied on the physical argument; this unorthodox approach may have its own technical merit.

Recalling symmetry property (3.15) and (3.16), one can supplement two more equations to the original simultaneous Wiener-Hopf equations,

$$\begin{cases} \tilde{c}_{44}a(\zeta)A_{sy}^{(sa)}(\zeta,p) + e_{15}e(\zeta)B_{sy}^{(sa)}(\zeta,p) = 0 \ , & (a) \\ a(\zeta)A_{sy}^{(sa)}(\zeta,p) = S_+(\zeta,p) \ , & (b) \\ e(\zeta)B_{sy}^{(sa)}(\zeta,p) = D_+(\zeta,p) \ , & (c) \\ \dfrac{e_{15}}{\epsilon_{11}^s}C_f W_{sy+}^{(sa)}(\zeta,p) + \Psi_{sy+}^{(sa)}(\zeta,p) = -\Big[\dfrac{g^*(p)}{p}\Big]\Big(\dfrac{e_{15}}{\epsilon_{11}^s}C_f\Big)\dfrac{w_0}{\zeta + s_s\cos\alpha} \ ; & (d) \end{cases} \tag{3.35}$$

where $S_+(\zeta,p), D_+(\zeta,p) \in \mathcal{O}(P_+)$.

Denote

$$a_+(\zeta) := \sqrt{s_s+\zeta} \ ; \ e_+(\zeta) := \sqrt{s_\ell+\zeta} \tag{3.36}$$

$$a_-(\zeta) := \sqrt{s_s-\zeta} \ ; \ e_-(\zeta) := \sqrt{s_\ell-\zeta} \tag{3.37}$$

From (3.35(b)) and (3.35(c)), one may have following Wiener-Hopf equations

$$a_-(\zeta)W_{sy+}^{(sa)}(\zeta,p) + a_-(\zeta)W_{sy-}^{(sa)}(\zeta,p) = S_+(\zeta,p)/a_+(\zeta) \tag{3.38}$$

$$e_-(\zeta)\Psi_{sy+}^{(sa)}(\zeta,p) + e_-(\zeta)\Psi_{sy-}^{(sa)}(\zeta,p) = D_+(\zeta,p)/e_+(\zeta) \tag{3.39}$$

In this particular problem, the above sectionally analytic functions may be viewed as meromorphic functions on the entire plane except possible branch cuts, say C_+ and C_- for instance, which are unknown for the moment; however, in this problem it is reasonable to expect these branch cuts reside along the real axis, i.e. $Im(\zeta) = 0$. Furthermore, from the standpoint of physics, the removable simple pole of the meromorphic function should be at $\zeta = -s_s \cos\alpha$, because the associated scattering field is driven by the incident acoustic wave. Then, from (3.40), for a fixed point $\zeta \in \mathbb{C}$ ($\mathbb{C} = P_+ \cup P_-$), it is not difficult to find that

$$W^{(sa)}_{sy-}(\zeta + i0, p) = W^{(sa)}_{sy+}(\zeta + i0, p)\Big(\frac{a_-(-\zeta_{s\alpha})}{a_-(\zeta + i0)} - 1\Big) \tag{3.40}$$

$$W^{(sa)}_{sy-}(\zeta - i0, p) = W^{(sa)}_{sy+}(\zeta - i0, p)\Big(\frac{a_-(-\zeta_{s\alpha})}{a_-(\zeta - i0)} - 1\Big) \tag{3.41}$$

where $i = \sqrt{-1}$. Eq. (3.40) and (3.41) are legitimate factorization, if ζ is away from the branch cut.

Thus, $\forall\ \zeta \in P_-$, one may find that

$$W^{(sa)}_{sy-}(\zeta, p) = \bar{W}^{(sa)}_{sy+}(\zeta, p)\Big(\frac{a_-(-\zeta_{s\alpha})}{a_-(\zeta)} - 1\Big) \tag{3.42}$$

$$\tilde{W}^{(sa)}_{sy+}(\zeta, p) = 0 \tag{3.43}$$

and $\forall\ \zeta \in P_+$

$$\bar{W}^{(sa)}_{sy-}(\zeta, p) = W^{(sa)}_{sy+}(\zeta, p)\Big[\frac{a_-(-\zeta_{s\alpha})}{2}\Big(\frac{1}{a^+_-(\zeta)} + \frac{1}{a^-_-(\zeta)}\Big) - 1\Big] \tag{3.44}$$

$$\tilde{W}^{(sa)}_{sy-}(\zeta, p) = W^{(sa)}_{sy+}(\zeta, p)\Big[\frac{a_-(-\zeta_{s\alpha})}{2}\Big(\frac{1}{a^+_-(\zeta)} - \frac{1}{a^-_-(\zeta)}\Big)\Big] \tag{3.45}$$

where the following notations are adopted

$$\bar{W}^{(sa)}_{sy\pm}(\zeta, p) := \frac{1}{2}\Big(W^{(sa)+}_{sy\pm}(\zeta, p) + W^{(sa)-}_{sy\pm}(\zeta, p)\Big) \tag{3.46}$$

$$\tilde{W}^{(sa)}_{sy\pm}(\zeta, p) := \frac{1}{2}\Big(W^{(sa)+}_{sy\pm}(\zeta, p) - W^{(sa)-}_{sy\pm}(\zeta, p)\Big) \tag{3.47}$$

$$W^{(sa)\pm}_{sy\pm}(\zeta, p) := W^{(sa)}_{sy\pm}(\zeta \pm i0, p) \tag{3.48}$$

$$a^\pm_\pm(\zeta) := a_\pm(\zeta \pm i0) \tag{3.49}$$

Note that by definition,

$$\begin{aligned}
W^{(sa)}_{sy-}(\zeta + i0, p) = W^{(sa)}_{sy-}(\zeta - i0, p) &= W^{(sa)}_{sy-}(\zeta, p)\ ,\ \forall\ \zeta \in P_- \\
a_-(\zeta + i0) = a_-(\zeta - i0) &= a_-(\zeta)\ , \qquad\qquad \forall\ \zeta \in P_- \\
W^{(sa)}_{sy+}(\zeta + i0, p) = W^{(sa)}_{sy+}(\zeta - i0, p) &= W^{(sa)}_{sy+}(\zeta, p)\ ;\ \forall\ \zeta \in P_+ \\
a_+(\zeta + i0) = a_+(\zeta - i0) &= a_+(\zeta)\ , \qquad\qquad \forall\ \zeta \in P_+
\end{aligned}$$

Similarly, based on (3.39) $\forall\ \zeta \in P_-$, one may find that

$$\Psi^{(sa)}_{sy-}(\zeta,p) = \bar{\Psi}^{(sa)}_{sy+}(\zeta,p)\Big(\frac{e_-(-\zeta_{s\alpha})}{e_-(\zeta)} - 1\Big) \tag{3.50}$$

$$\tilde{\Psi}^{(sa)}_{sy+}(\zeta,p) = 0 \tag{3.51}$$

and $\forall\ \zeta \in P_+$

$$\bar{\Psi}^{(sa)}_{sy-}(\zeta,p) = \Psi^{(sa)}_{sy+}(\zeta,p)\Big[\frac{e_-(-\zeta_{s\alpha})}{2}\Big(\frac{1}{e^+_-(\zeta)} + \frac{1}{e^-_-(\zeta)}\Big) - 1\Big] \tag{5.32}$$

$$\tilde{\Psi}^{(sa)}_{sy-}(\zeta,p) = \Psi^{(sa)}_{sy+}(\zeta,p)\Big[\frac{e_-(-\zeta_{s\alpha})}{2}\Big(\frac{1}{e^+_-(\zeta)} - \frac{1}{e^-_-(\zeta)}\Big)\Big] \tag{3.53}$$

where

$$\bar{\Psi}^{(sa)}_{sy\pm}(\zeta,p) := \frac{1}{2}\Big(\Psi^{(sa)+}_{sy\pm}(\zeta,p) + \Psi^{(sa)-}_{sy\pm}(\zeta,p)\Big) \tag{3.54}$$

$$\tilde{\Psi}^{(sa)}_{sy\pm}(\zeta,p) := \frac{1}{2}\Big(\Psi^{(sa)+}_{sy\pm}(\zeta,p) - \Psi^{(sa)-}_{sy\pm}(\zeta,p)\Big) \tag{3.55}$$

$$\Psi^{(sa)\pm}_{sy\pm}(\zeta,p) := \Psi^{(sa)}_{sy\pm}(\zeta \pm i0,p) \tag{3.56}$$

$$e^\pm_\pm(\zeta) := e_\pm(\zeta \pm i0) \tag{3.57}$$

Furthermore, from (3.35(a)), one can derive that $\forall\ \zeta \in P_-$

$$\begin{aligned}&\tilde{c}_{44}a_-(\zeta)a^\pm_+(\zeta)W^{(sa)\pm}_{sy+}(\zeta,p) + \tilde{c}_{44}a_-(\zeta)a^\pm_+(\zeta)W^{(sa)}_{sy-}(\zeta,p)\\ &+e_{15}e_-(\zeta)e^\pm_+(\zeta)\Psi^{(sa)\pm}_{sy+}(\zeta,p) + e_{15}e_-(\zeta)e^\pm_+(\zeta)\Psi^{(sa)}_{sy-}(\zeta,p) = 0\end{aligned} \tag{3.58}$$

which implies that

$$\begin{aligned}&\pm\tilde{c}_{44}a_-(\zeta)a^\pm_+(\zeta)\tilde{W}^{(sa)}_{sy+}(\zeta,p) + \tilde{c}_{44}a_-(-\zeta_{s\alpha})a^\pm_+(\zeta)\bar{W}^{(sa)}_{sy+}(\zeta,p)\\ &\pm e_{15}e_-(\zeta)e^\pm_+(\zeta)\tilde{\Psi}^{(sa)}_{sy+}(\zeta,p) + e_{15}e_-(-\zeta_{s\alpha})e^\pm_+(\zeta)\bar{\Psi}^{(sa)}_{sy+}(\zeta,p) = 0\end{aligned} \tag{3.59}$$

Consider the fact that $\tilde{W}^{(sa)}_{sy+}(\zeta,p) = \tilde{\Psi}^{(sa)}_{sy+}(\zeta,p) = 0$ and let

$$\bar{a}_+(\zeta) := \frac{1}{2}\Big(a^+_+(\zeta) + a^-_+(\zeta)\Big) \tag{3.60}$$

$$\bar{e}_+(\zeta) := \frac{1}{2}\Big(e^+_+(\zeta) + e^-_+(\zeta)\Big) \tag{3.61}$$

We end with

$$\tilde{c}_{44}a_-(-\zeta_{s\alpha})\bar{a}_+(\zeta)\bar{W}^{(sa)}_{sy+}(\zeta,p)+e_{15}e_-(-\zeta_{s\alpha})\bar{e}_+(\zeta)\bar{\Psi}^{(sa)}_{sy+}(\zeta,p)=0 \tag{3.62}$$

Rewrite (3.35(d)) as

$$\Big(\frac{e_{15}}{\epsilon^s_{11}}C_f\Big)\bar{W}^{(sa)}_{sy+}(\zeta,p)+\bar{\Psi}^{(sa)}_{sy+}(\zeta,p)=-\Big(\frac{e_{15}}{\epsilon^s_{11}}C_f\Big)\Big[\frac{g^*(p)}{p}\Big]\frac{w_0}{\zeta+\zeta_{s\alpha}} \tag{3.63}$$

Solving (3.62), (3.63) together, we obtain the the following closed form solutions

$$\bar{W}^{(sa)}_{sy+}(\zeta,p)=\Big[\frac{g^*(p)}{p}\Big]\Big(\frac{w_0}{\zeta+\zeta_{s\alpha}}\Big)\frac{k_e^2e_-(-\zeta_{s\alpha})\bar{e}_+(\zeta)}{\bar{\Delta}^{(sa)}(\zeta)} \tag{3.64}$$

$$\bar{\Psi}^{(sa)}_{sy+}(\zeta,p)=-\Big[\frac{g^*(p)}{p}\Big]\Big(\frac{e_{15}}{\epsilon^s_{11}}C_f\Big)\Big(\frac{w_0}{\zeta+\zeta_{s\alpha}}\Big)\frac{a_-(-\zeta_{s\alpha})\bar{a}_+(\zeta)}{\bar{\Delta}^{(sa)}(\zeta)} \tag{3.65}$$

where $\bar{\Delta}^{(sa)}(\zeta):=a_-(-\zeta_{s\alpha})\bar{a}_+(\zeta)-k_e^2e_-(-\zeta_{s\alpha})\bar{e}_+(\zeta)$.

Again considering the fact that $\tilde{W}^{(sa)}_{sy+}(\zeta,p)=\tilde{\Psi}^{(sa)}_{sy+}(\zeta,p)=0$, one can conclude that $\forall\ \zeta\in P_-$,

$$W^{(sa)}_{sy+}(\zeta+i0,p)=W^{(sa)}_{sy+}(\zeta-i0,p)=\bar{W}^{(sa)}_{sy+}(\zeta,p) \tag{3.66}$$

$$\Psi^{(sa)}_{sy+}(\zeta+i0,p)=\Psi^{(sa)}_{sy+}(\zeta-i0,p)=\bar{\Psi}^{(sa)}_{sy+}(\zeta,p) \tag{3.67}$$

Consequently, $\forall\ \zeta\in P_-$,

$$W^{(sa)}_{sy-}(\zeta,p)=W^{(sa)}_{sy+}(\zeta,p)\Big(\frac{a_-(-\zeta_{s\alpha})}{a_-(\zeta)}-1\Big) \tag{3.68}$$

$$\Psi^{(sa)}_{sy-}(\zeta,p)=\Psi^{(sa)}_{sy+}(\zeta,p)\Big(\frac{e_-(-\zeta_{s\alpha})}{e_-(\zeta)}-1\Big) \tag{3.69}$$

Then, the symmetry solutions in P_- are attainable, i.e. $\forall\ \zeta\in P_-$,

$$\boxed{\begin{aligned} A^{(sa)}_{sy}(\zeta,p)&=\Big[\frac{g^*(p)}{p}\Big]\Big(\frac{w_0}{\zeta+\zeta_{s\alpha}}\Big)\Big(\frac{a_-(-\zeta_{s\alpha})}{a_-(\zeta)}\Big)\frac{k_e^2e_-(-\zeta_{s\alpha})\bar{e}_+(\zeta)}{\bar{\Delta}^{(sa)}(\zeta)}\ , && (a)\\ B^{(sa)}_{sy}(\zeta,p)&=-\Big(\frac{e_{15}}{\epsilon^s_{11}}C_f\Big)\Big[\frac{g^*(p)}{p}\Big]\Big(\frac{w_0}{\zeta+\zeta_{s\alpha}}\Big)\Big(\frac{e_-(-\zeta_{s\alpha})}{e_-(\zeta)}\Big)\frac{a_-(-\zeta_{s\alpha})\bar{a}_+(\zeta)}{\bar{\Delta}^{(sa)}(\zeta)}\ , && (b)\end{aligned}} \tag{3.70}$$

Technically speaking, to this end, the symmetric part of Wiener-Hopf equations is considered being solved, since in the ensuing inversion process, one only needs the information of $A^{(sa)}_{sy}(\zeta,p)$ and $B^{(sa)}_{sy}(\zeta,p)$ $\forall\ \zeta\in P_-$.

It is, however, quaint to show that (3.70(a)(b)) are also valid $\forall\ \zeta \in P_+$. To do so, we seek the expressions of sectionally analytic functions, $W^{(sa)}_{sy+}(\zeta,p)$ and $W^{(sa)}_{sy+}(\zeta,p)$, in P_+ From Eq. (3.44)–(3.45) and (3.52)–(3.53), after some algebraic manipulation, one can drive that $\forall\ \zeta \in P_+$,

$$W^{(sa)+}_{sy-}(\zeta,p) = W^{(sa)}_{sy+}(\zeta,p)\Big(\frac{a_-(-\zeta_{s\alpha})}{a^+_-(\zeta)} - 1\Big) \tag{3.71}$$

$$W^{(sa)-}_{sy-}(\zeta,p) = W^{(sa)}_{sy+}(\zeta,p)\Big(\frac{a_-(-\zeta_{s\alpha})}{a^-_-(\zeta)} - 1\Big) \tag{3.72}$$

and

$$\Psi^{(sa)+}_{sy-}(\zeta,p) = \Psi^{(sa)}_{sy+}(\zeta,p)\Big(\frac{e_-(-\zeta_{s\alpha})}{e^+_-(\zeta)} - 1\Big) \tag{3.73}$$

$$\Psi^{(sa)-}_{sy-}(\zeta,p) = \Psi^{(sa)}_{sy+}(\zeta,p)\Big(\frac{e_-(-\zeta_{s\alpha})}{e^-_-(\zeta)} - 1\Big) \tag{3.74}$$

Utilizing (3.35(a)), one can show that $\forall\ \zeta \in P_+$

$$\begin{aligned}&\tilde{c}_{44}a^{\pm}_-(\zeta)a_+(\zeta)W^{(sa)}_{sy+}(\zeta,p) + \tilde{c}_{44}a^{\pm}_-(\zeta)a_+(\zeta)W^{(sa)\pm}_{sy-}(\zeta,p)\\ &+e_{15}e^{\pm}_-(\zeta)e_+(\zeta)\Psi^{(sa)}_{sy+}(\zeta,p) + e_{15}e^{\pm}_-(\zeta)e_+(\zeta)\Psi^{(sa)\pm}_{sy-}(\zeta,p) = 0\end{aligned} \tag{3.75}$$

Substituting (3.71), (3.73) into (3.75) yields

$$\tilde{c}_{44}a_-(-\zeta_{s\alpha})a_+(\zeta)W^{(sa)}_{sy+}(\zeta,p) + e_{15}e_-(-\zeta_{s\alpha})e_+(\zeta)\Psi^{(sa)}_{sy+}(\zeta,p) = 0 \tag{3.76}$$

Solving (3.76) and (3.35(d)) together, we find that

$$\boxed{\begin{aligned}W^{(sa)}_{sy+}(\zeta,p) &= \Big[\frac{g^*(p)}{p}\Big]\Big(\frac{w_0}{\zeta+\zeta_{sa}}\Big)\frac{k^2_e e_-(-\zeta_{s\alpha})e_+(\zeta)}{\Delta^{(sa)}(\zeta)}\ , & (a)\\ \Psi^{(sa)}_{sy+}(\zeta,p) &= -\Big(\frac{e_{15}}{\epsilon^s_{11}}C_f\Big)\Big[\frac{g^*(p)}{p}\Big]\Big(\frac{w_0}{\zeta+\zeta_{sa}}\Big)\frac{a_-(-\zeta_{s\alpha})a_+(\zeta)}{\Delta^{(sa)}(\zeta)}\ , & (b)\end{aligned}} \tag{3.77}$$

where $\Delta^{(sa)}(\zeta) := a_-(-\zeta_{s\alpha})a_+(\zeta) - k^2_e e_-(-\zeta_{s\alpha})e_+(\zeta)$. As a matter of fact, one may anticipate the results from (3.66) and (3.67) by extrapolation.

Remark 3.33. a. It seems that there is another simple pole in the expressions of (3.77). Indeed, there is a "simple pole" at

$$\zeta_p = -\frac{[s^2_s(1+\cos\alpha) - k^4_e s_\ell(s_\ell + s_s\cos\alpha)]}{[s_s(1+\cos\alpha) - k^4_e(s_\ell + s_s\cos\alpha)]}$$

Since $k_e^4 < 1$, $s_\ell << s_s$, it leads to $\zeta_p < 0$; in other words, ζ_p is not a simple pole for $W^{(sa)}_{sy+}(\zeta,p)$ and $\Psi^{(sa)}_{sy+}(\zeta,p)$, because $\zeta_p \notin P_+$. And $\forall\ \zeta \in P_-$, $W^{(sa)}_{sy+}(\zeta,p)$ and $\Psi^{(sa)}(\zeta,p)$ are given by (3.66) and (3.67) instead of (3.77).

b. From (3.70(a)) and (3.70(b)), obviously, $W^{(sa)}_{sy-}(\zeta,p) \neq 0$, $\Psi^{(sa)}_{sy-}(\zeta,p) \neq 0$, which then predict non-zero bulk scattering fields at the trail of the crack, or tip of the crack. This is one of unique feature in scattering of electroacoustic waves. □

(c)Solutions of electric incident waves

Consider only the electric incident wave, i.e. $w_0 = 0; \psi_0 \neq 0$. The boundary conditions are

$$\begin{cases} \bar{c}_{44}\dfrac{\partial w^{(se)}}{\partial x_2} + e_{15}\dfrac{\partial \psi^{(se)}}{\partial x_2} = -e_{15}\dfrac{\partial \psi^{(i)}}{\partial x_2} \\ \dfrac{e_{15}}{\epsilon^s_{11}} C_f w^{(se)} + \psi^{(se)} = -\psi^{(i)} \end{cases} \quad 0 < x_1 < \infty\ ,\ x_2 = 0\ ; \tag{3.78}$$

Again, the transformed solutions are split into two parts: anti-symmetry part and symmetry part, namely,

$$\begin{cases} \hat{w}^{(se)*}(\zeta,x_2,p) = \Big(A^{(se)}_{an}(\zeta,p)\,\mathrm{sgn}(x_2) + A^{(se)}_{sy}(\zeta,p)\Big)\exp\big(-pa(\zeta)|x_2|\big) \\ \hat{\psi}^{(se)*}(\zeta,x_2,p) = \Big(B^{(se)}_{an}(\zeta,p)\,\mathrm{sgn}(x_2) + B^{(se)}_{sy}(\zeta,p)\Big)\exp\big(-pe(\zeta)|x_2|\big) \end{cases} \tag{3.79}$$

The Wiener-Hopf equations for anti-symmetry solutions are

$$\begin{cases} -\bar{c}_{44}a(\zeta)A^{(se)}_{an}(\zeta,p) - e_{15}e(\zeta)B^{(se)}_{an}(\zeta,p) = \dfrac{\Sigma^{(se)}_{-}(\zeta,p)}{p} + \Big[\dfrac{g^*(p)}{p}\Big]\dfrac{e_{15}s_\ell\sin\alpha\psi_0}{\zeta + s_\ell\cos\alpha} \\ \dfrac{e_{15}}{\epsilon^s_{11}} C_f W^{(se)}_{an+}(\zeta,p) + \Psi^{(se)}_{an+}(\zeta,p) = 0 \end{cases} \tag{3.80}$$

where

$$\Sigma^{(se)}_{-}(\zeta,p) := \int_{-\infty}^{0}\int_{0}^{\infty} \sigma^{(se)}_{23}(x_1,+0,t)\exp\Big(-p[t+\zeta x_1]\Big)dtdx_1$$

Let $\zeta_{\ell\alpha} := s_\ell\cos\alpha$. The anti-symmetry solutions are as follows

$$\boxed{\begin{aligned} A^{(se)}_{an}(\zeta,p) &= -\Big[\frac{g^*(p)}{p}\Big]\Big(\frac{s_\ell\psi_0\sin\alpha}{1-k_e^2}\Big)\frac{M_-(-\zeta_{\ell\alpha})\mathcal{D}_+(\zeta)\sqrt{s_\ell+\zeta}}{(s_{bge}+\zeta)(\zeta+\zeta_{\ell\alpha})}\ , & (a) \\ B^{(se)}_{an}(\zeta,p) &= \Big[\frac{g^*(p)}{p}\Big]\Big(\frac{k_e^2 s_\ell\psi_0\sin\alpha}{1-k_e^2}\Big)\frac{M_-(-\zeta_{\ell\alpha})\mathcal{D}_+(\zeta)\sqrt{s_\ell+\zeta}}{(s_{bge}+\zeta)(\zeta+\zeta_{\ell\alpha})}\ , & (b) \\ \Sigma^{(se)}_{-}(\zeta,p) &= e_{15}g^*(p)\Big(\frac{s_\ell\psi_0\sin\alpha}{\zeta+\zeta_{\ell\alpha}}\Big)\Big[\frac{M_-(-\zeta_{\ell\alpha})}{M_-(\zeta)} - 1\Big]\ , & (c) \end{aligned}}$$

(3.81)

On the other hand, the Wiener-Hopf equations for symmetry solutions are

$$\begin{cases} -\bar{c}_{44}a(\zeta)A^{(se)}_{sy}(\zeta,p) - e_{15}e(\zeta)B^{(se)}_{sy}(\zeta,p) = 0 \\ \dfrac{e_{15}}{\epsilon^s_{11}}C_f W^{(se)}_{sy+}(\zeta,p) + \Psi^{(se)}_{sy+}(\zeta,p) = -\Big[\dfrac{g^*(p)}{p}\Big]\dfrac{\psi_0}{\zeta + s_\ell\cos\alpha} \end{cases} \tag{3.82}$$

The corresponding symmetry solutions are

$$\boxed{\begin{aligned} A^{(se)}_{sy}(\zeta,p) &= \Big[\frac{g^*(p)}{p}\Big]\Big(\frac{e_{15}}{\epsilon^s_{11}}\Big)\Big(\frac{\psi_0}{\zeta+\zeta_{\ell\alpha}}\Big)\Big(\frac{a_-(-\zeta_{\ell\alpha})}{a_-(\zeta)}\Big)\frac{e_-(-\zeta_{\ell\alpha})\bar{e}_+(\zeta)}{\bar{\Delta}^{(se)}(\zeta)} \quad (a) \\ B^{(se)}_{sy}(\zeta,p) &= -\Big[\frac{g^*(p)}{p}\Big]\Big(\frac{\psi_0}{\zeta+\zeta_{\ell\alpha}}\Big)\Big(\frac{e_-(-\zeta_{\ell\alpha})}{e_-(\zeta)}\Big)\frac{a_-(-\zeta_{\ell\alpha})\bar{a}_+(\zeta)}{\bar{\Delta}^{(se)}(\zeta)} \quad (b) \end{aligned}}$$

(3.83)

where and in sequel

$$\bar{\Delta}^{(se)}(\zeta) := a_-(-\zeta_{\ell\alpha})\bar{a}_+(\zeta) - k_e^2 e_-(-\zeta_{\ell\alpha})\bar{e}_+(\zeta) \tag{3.84}$$

Remark 3.4. Let the piezoelectric coefficient $e_{15} \to 0$. From (3.83(a)) and (3.83(b)), one will find that

$$A^{(se)}_{sy}(\zeta,p) \to 0 \tag{3.85}$$

$$B^{(se)}_{sy}(\zeta,p) \to -\Big[\frac{g^*(p)}{p}\Big]\Big(\frac{\psi_0}{\zeta + s_\ell\cos\alpha}\Big)\frac{e_-(\zeta_{\ell\alpha})}{e_-(\zeta)} \tag{3.86}$$

Eq.(3.86) corresponds to the classical result obtained by Jones [1952] for the Sommerfeld problem. □

4. Exact inversions

In this section, the drived transformed solutions are converted back to physical space by exact inversion. The main technical ingredients for exact inversions are the ingenious Cagniard-de Hoop technique (Cagniard [1939], de Hoop [1960]), and Cauchy residual theorem. A remarkable fact about diffraction problems in piezoelectric field is the unusual scattering patterns induced by acoustic source, which are not only different from the scattering fields in purely elastic medium by the same acoustic source, but also different from those in the same piezoelectric medium by the electric incident wave. To compare the differences between the two, a detailed exposition is presented as follows.

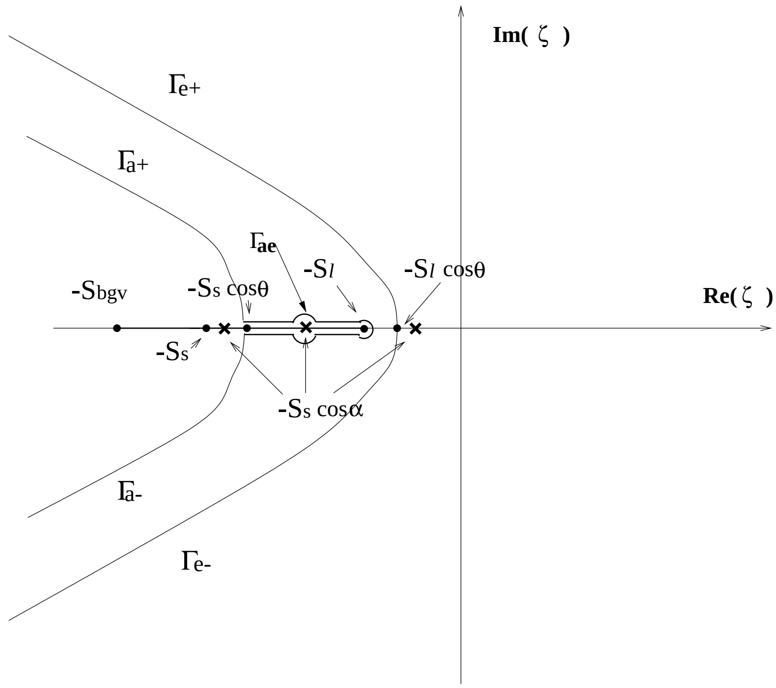

Figure 2.
Cagniard-de Hoop inversion pathes for acoustic excitation

(a) Scattering fields generated by acoustic source

Consider the displacement and electric potential scattering fileds under the acoustic excitation,

$$\begin{aligned} w^{*(sa)}(x_1,x_2,p) = {} & \frac{g^*(p)}{2\pi i}\int_{\zeta_a-i\infty}^{\zeta_a+i\infty}\Big(\frac{w_0}{\zeta+\zeta_{s\alpha}}\Big) \\ & \Big\{-\frac{s_s\sin\alpha M_-(-\zeta_{s\alpha})}{(1-k_e^2)}\frac{\sqrt{s_s+\zeta}}{(s_{bge}+\zeta)}\mathcal{D}_+(\zeta)sgn(x_2)+\frac{a_-(-\zeta_{s\alpha})}{a_-(\zeta)} \\ & \cdot\frac{k_e^2 e_-(-\zeta_{s\alpha})\bar{e}_+(\zeta)}{\bar{\Delta}^{(sa)}(\zeta)}\Big\}\exp\Big(-p[a(\zeta)|x_2|-\zeta x_1]\Big)d\zeta \end{aligned} \tag{4.1}$$

$$\begin{aligned} \phi^{*(sa)}(x_1,x_2,p) = {} & \frac{g^*(p)}{2\pi i}\Big(\frac{e_{15}}{\epsilon_{11}^s}C_f\Big)\Big\{\int_{\zeta_a-i\infty}^{\zeta_a+i\infty}\Big(\frac{w_0}{\zeta+\zeta_{s\alpha}}\Big) \\ & \Big(-\frac{s_s\sin\alpha M_-(-\zeta_{s\alpha})}{(1-k_e^2)}\frac{\sqrt{s_s+\zeta}}{(s_{bge}+\zeta)}\mathcal{D}_+(\zeta)sgn(x_2) \\ & +\frac{a_-(-\zeta_{s\alpha})}{a_-(\zeta)}\frac{k_e^2 e_-(-\zeta_{s\alpha})\bar{e}_+(\zeta)}{\bar{\Delta}^{(sa)}(\zeta)}\Big)\exp\Big(-p[a(\zeta)|x_2|-\zeta x_1]\Big)d\zeta \end{aligned}$$

$$
+\int_{\zeta_e-i\infty}^{\zeta_e+i\infty}\Big(\frac{w_0}{\zeta+\zeta_{s\alpha}}\Big)\Big(\frac{s_s\sin\alpha M_-(-\zeta_{s\alpha})}{(1-k_e^2)}\frac{\sqrt{s_s+\zeta}}{(s_{bge}+\zeta)}\mathcal{D}_+(\zeta)\,\mathrm{sgn}(x_2)
- \frac{e_-(-\zeta_{s\alpha})}{e_-(\zeta)}\frac{a_-(-\zeta_{s\alpha})\bar{a}_+(\zeta)}{\bar{\Delta}^{(sa)}(\zeta)}\Big)\exp\Big(-p[e(\zeta)|x_2|-\zeta x_1]\Big)d\zeta\Big\} \tag{4.2}
$$

where $\varepsilon_+ < \zeta_a, \zeta_e < \varepsilon_-$.

Shown in Figure (2), three different inversion pathes are chosen: $\Gamma_a, \Gamma_{ae}, \Gamma_e$, in which

$$
\begin{aligned}
a(\zeta)x_2-\zeta x_1 &= t\ ,\ \zeta\in\Gamma_a,\Gamma_{ae}\\
e(\zeta)x_2-\zeta x_1 &= t\ ,\ \zeta\in\Gamma_e
\end{aligned} \tag{4.3}
$$

Let $x_1 = r\cos\theta$, $x_2 = r\sin\theta$. One then has

$$
\zeta_{a\pm} = \frac{1}{r}\Big[-t\cos\theta \pm i\sin\theta\sqrt{t^2-s_s^2r^2}\Big]\ ,\ s_sr\le t<\infty \tag{4.4}
$$

$$
\zeta_{ae\pm} = \frac{1}{r}\Big[-t\cos\theta \pm \sin\theta\sqrt{s_s^2r^2-t^2}\Big]\pm i\epsilon\ ,\ t_{ae}\le t< s_sr \tag{4.5}
$$

$$
\zeta_{e\pm} = \frac{1}{r}\Big[-t\cos\theta \pm i\sin\theta\sqrt{t^2-s_\ell^2r^2}\Big]\ ,\ s_\ell r\le t\le\infty \tag{4.6}
$$

where $t_{ae} = \sqrt{s_s^2-s_\ell^2}x_2 + s_\ell x_1$.

It should be noted that at $\zeta = -s_s\cos\theta$ path Γ_a intercepts the real axis $Re(\zeta)$. Thus, a supplement path Γ_{ae} is needed to circumvent the branch cut of multivalued function $e(\zeta) = \sqrt{s_\ell^2-\zeta^2}$. This will lead to the occurrence of an electroacoustic head wave (see discussions in Li [1998] as well as Lin et. al [1989]). Along path Γ_{ae}, the parameter θ varies in the range,

$$
0\le\theta\le\theta_{cr}^{ae}\ ,\ \text{or}\ \pi\le\theta\le\pi-\theta_{cr}^{ae} \tag{4.7}
$$

where $\theta_{cr}^{ae} := \cos^{-1}\Big(s_\ell/s_s\Big)$.

Following de Hoop (1969), one may show that

$$
\frac{\partial\zeta_{a\pm}}{\partial t} = \frac{\pm ia(\zeta_{a\pm})}{\sqrt{t^2-s_s^2r^2}}\ ;\ a(\zeta_{a\pm}) = \frac{\sin\theta}{r}t\pm i\frac{\cos\theta}{r}\sqrt{t^2-s_s^2r^2}\ ; \tag{4.8}
$$

$$
\frac{\partial\zeta_{ae\pm}}{\partial t} = \frac{\mp a(\zeta_{ae\pm})}{\sqrt{s_s^2r^2-t^2}}\ ;\ a(\zeta_{ae\pm}) = \frac{\sin\theta}{r}t\pm \frac{\cos\theta}{r}\sqrt{s_s^2r^2-t^2}\ ; \tag{4.9}
$$

$$
\frac{\partial\zeta_{e\pm}}{\partial t} = \frac{\pm ie(\zeta_{e\pm})}{\sqrt{t^2-s_\ell^2r^2}}\ ;\ e(\zeta_{e\pm}) = \frac{\sin\theta}{r}t\pm i\frac{\cos\theta}{r}\sqrt{t^2-s_\ell^2r^2}\ ; \tag{4.10}
$$

and subsequently exact inversions are found

$$w^{(sa)}(x_1,x_2,t) = \int_0^t g(t-\tau)w_\delta^{(sa)}(x_1,x_2,\tau)d\tau + w_{ra}(x_1,x_2,t) \tag{4.11}$$

$$\phi^{(sa)}(x_1,x_2,t) = \int_0^t g(t-\tau)\phi_\delta^{(sa)}(x_1,x_2,\tau)d\tau + \phi_{ra}(x_1,x_2,t) \tag{4.12}$$

where the subscript "δ" represents the scattering fields due to the impulsive incident wave, and w_{ra}, ϕ_{ra} are geometrical diffraction (reflection & refraction) fields with respect to displacement and electric potential, in which the subscript "ra" stands for reflection/refraction field due to acoustic source.

The formal solutions for incident pulse plane wave are

$$\begin{aligned} w_\delta^{(sa)}(x_1,x_2,t) = \frac{w_0}{\pi}\Bigg\{ & Re\Big[\Big(\frac{a(\zeta)}{(\zeta+\zeta_{s\alpha})\sqrt{t^2-s_s^2r^2}}\Big) \\ & \Big(-\frac{s_s\sin\alpha M_-(-\zeta_{s\alpha})}{(1-k_e^2)}\frac{\sqrt{s_s+\zeta}}{(s_{bge}+\zeta)}\mathcal{D}_+(\zeta)\,\mathrm{sgn}(x_2) \\ & + \frac{a_-(-\zeta_{s\alpha})}{a_-(\zeta)}\frac{k_e^2 e_-(-\zeta_{s\alpha})\bar{e}_+(\zeta)}{\bar{\Delta}^{(sa)}(\zeta)}\Big)\Big]\Big|_{\zeta\in\Gamma_{a+}} H(t-s_sr) \\ & + Im\Bigg[\Big(\frac{s_s\sin\alpha M_-(-\zeta_{s\alpha})a(\zeta)}{(\zeta+\zeta_{s\alpha})\sqrt{s_s^2r^2-t^2}}\Big)\Big[\frac{\mathcal{D}_+(\zeta)}{(1-k_e^2)}\frac{\sqrt{s_s+\zeta}}{(s_{bge}+\zeta)}\Big]\Bigg]\Big|_{\zeta\in\Gamma_{ae+}} \\ & \cdot\,\mathrm{sgn}(x_2)\Big(H(t-t_{ae})-H(t-s_sr)\Big)\Bigg\} \end{aligned} \tag{4.13}$$

$$\begin{aligned} \phi_\delta^{(sa)}(x_1,x_2,t) = \Big(\frac{e_{15}}{\epsilon_{11}^s}C_f\frac{w_0}{\pi}\Big)\Bigg\{ & Re\Big[\Big(\frac{a(\zeta)}{(\zeta+\zeta_{s\alpha})\sqrt{t^2-s_s^2r^2}}\Big) \\ & \Big(-\frac{s_s\sin\alpha M_-(-\zeta_{s\alpha})}{(1-k_e^2)}\frac{\sqrt{s_s+\zeta}}{(s_{bge}+\zeta)}\mathcal{D}_+(\zeta)\,\mathrm{sgn}(x_2) + \frac{a_-(-\zeta_{s\alpha})}{a_-(\zeta)} \\ & \cdot\frac{k_e^2 e_-(-\zeta_{s\alpha})\bar{e}_+(\zeta)}{\bar{\Delta}^{(sa)}(\zeta)}\Big)\Big]\Big|_{\zeta\in\Gamma_{a+}} H(t-s_sr) \\ & + Im\Bigg[\Big(\frac{s_s\sin\alpha M_-(-\zeta_{s\alpha})a(\zeta)}{(\zeta+\zeta_{s\alpha})\sqrt{s_s^2r^2-t^2}}\Big)\Big[\frac{\mathcal{D}_+(\zeta)}{(1-k_e^2)}\frac{\sqrt{s_s+\zeta}}{(s_{bge}+\zeta)}\Big]\Bigg]\Big|_{\zeta\in\Gamma_{ae+}} \\ & \mathrm{sgn}(x_2)\Big(H(t-t_{ae})-H(t-s_sr)\Big) \\ & + Re\Big[\Big(\frac{e(\zeta)}{(\zeta+\zeta_{s\alpha})\sqrt{t^2-s_\ell^2r^2}}\Big)\Big(\frac{s_s\sin\alpha M_-(-\zeta_{s\alpha})}{(1-k_e^2)}\frac{\sqrt{s_s+\zeta}}{(s_{bge}+\zeta)}\mathcal{D}_+(\zeta) \end{aligned}$$

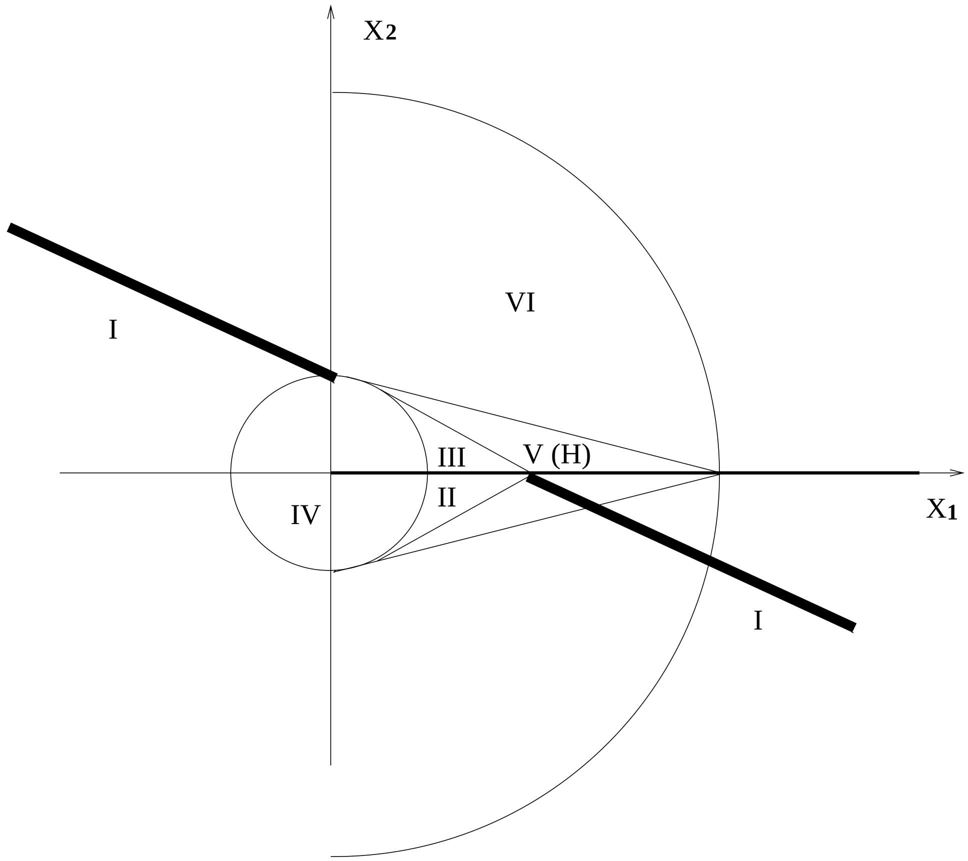

Figure 3.
The diffraction patterns excited by acoustic incident wave (I): $\alpha < \cos^{-1}(s_\ell/s_s)$.

$$\mathrm{sgn}(x_2) - \frac{e_-(-\zeta_{s\alpha})}{e_-(\zeta)}\frac{a_-(-\zeta_{s\alpha})\bar{a}_+(\zeta)}{\bar{\Delta}^{(sa)}(\zeta)}\Big)\Big]\Big|_{\zeta\in\Gamma_{e+}} H(t - s_\ell r)\Big\} \qquad (4.14)$$

Remark 4.1. **1.** Along the supplement path, Γ_{ae}, the phase function, $\mathcal{D}_+(\zeta)$ is complex and multi-valued, as shown in Figure (2).

2. The terms activating during the period, $t_{ae} \le t \le s_s r$, are the electroacoustic head waves. Because $\bar{e}(\zeta) = 0 \ \forall \zeta \in \Gamma_{ae}$, the contribution from the symmetry displacement solution to electroacoustic head wave is zero; in other words, the electroacoustic head waves solely come from the anti-symmetry solutions. □

In diffraction theory, both acoustic and electromagnetic, the simple pole that represents the incident source determines the geometrical reflection/refraction fields. These geometrical scattering patterns induced by acoustic excitation depend on the incident angle of the acoustic wave, because the position of simple pole relies on the angle of incident acoustic wave. Figure (2) shows that there are three different positions of simple pole, $\zeta = -\zeta_{s\alpha} = -s_s \cos\alpha$; and the positions of $-\zeta_{s\alpha}$ in ζ plane will directly affect the outcome of reflection/refraction fields. There are basically two cases:

(1) $\alpha < \cos^{-1}\Big(s_\ell/s_s\Big)$.

In this case,

$$Re\Big(\mathcal{D}_-(-\zeta_{s\alpha})\mathcal{D}_+(-\zeta_{s\alpha})\Big) = \frac{1}{1+k_e^2}$$

We have

$$w_{ra}(x_1,x_2,t) = \begin{cases} -w_0\dfrac{g(t-s_s[\sin\alpha x_2+\cos\alpha x_1])}{s_{bge}^2 - s_s^2\cos^2\alpha}\Big[\dfrac{s_s^2\sin^2\alpha}{(1-k_e^4)}\Big] & 0 \le \theta < \alpha \\ 0 & \alpha \le \theta < \pi \\ 0 & \pi \le \theta < \pi-\alpha \\ w_0\dfrac{g(t+s_s[\sin\alpha x_2-\cos\alpha x_1])}{s_{bge}^2 - s_s^2\cos^2\alpha}\Big[\dfrac{s_s^2\sin^2\alpha}{(1-k_e^4)}\Big] & \pi-\alpha \le \theta < 2\pi \end{cases} \tag{4.15}$$

and

$$\phi_{ra}(x_1,x_2,t) = \frac{e_{15}}{\epsilon_{11}^s}C_f w_{ra}(x_1,x_2,t) \tag{4.16}$$

Note that first, since $\bar{e}_+(-\zeta_{s\alpha}) = 0$ when $\alpha < \cos^{-1}(s_\ell/s_s)$, there is no contribution from the symmetry part of displacement solution; second, when $\alpha < \cos^{-1}\Big(s_\ell/s_s\Big)$, the simple pole, $\zeta = -s_s\cos\alpha$, is always at the left of integration contour, $\Gamma_{e\pm}$; therefore in Eq. (4.16), there is no contribution from the the pseudo electric potential, ψ, i.e. $\psi_{ra}(x_1,x_2,t) = 0$ in this case. The complete scattering pattern is shown in Figure (3), in which

I:Incident acoustic wave zone;
II: Electroacoustic wave reflection zone;
III: Electroacoustic wave refraction zone;
IV: Electroacoustic wave scattering zone;
V:Electroacoustic head wave zone;
VI: Electric wave scattering zone.

Since in general the transmission coefficient along the slit

$$T(k_e) := 1 - \frac{s_s^2\sin^2\alpha}{(1-k_e^4)(s_{bge}^2 - s_s^2\cos^2\alpha)} \neq 0\ , \tag{4.17}$$

which implies that there is no shadow zone above the traction free/perfectly conductive slit, instead there is a refraction zone. In contrast to the purely acoustic

SH wave diffraction by a half-plane, one can see that the traction free/perfectly conductive half-plane is somewhat transparent to acoustic incident wave, because of the electro-mechanical coupling effect. If $k_e \to 0$, the shadow zone reappears because $T(k_e) \to 0$.

(2) $\alpha \geq \cos^{-1}\Bigl(s_\ell/s_s\Bigr)$.

In this case, $\bar{e}_+(-\zeta_{s\alpha}) = \sqrt{s_\ell(1-\cos\beta)} \neq 0$. Thus, there is a non-zero contribution from the symmetry displacement solution to the geometrical diffraction fields. Let $\beta := \cos^{-1}\Bigl(\tau_\ell^{-1}\cos\alpha\Bigr)$, $\tau_\ell := s_\ell/s_s$. We have

$$w_{ra}(x_1,x_2,t) = \begin{cases} w_0 g(t - s_s[\sin\alpha x_2 + \cos\alpha x_1])\Bigl[-\dfrac{s_s^2\sin^2\alpha}{(1-k_e^2)} & \\ \cdot\dfrac{\mathcal{D}_-(-\zeta_{s\alpha})\mathcal{D}_+(-\zeta_{s\alpha})}{s_{bge}^2 - s_s^2\cos^2\alpha} + \dfrac{k_e^2 s_\ell\sin\beta}{\bar{\Delta}^{(sa)}(-\zeta_{s\alpha})}\Bigr] & 0 \leq \theta < \alpha \\ 0 & \alpha < \theta \leq \pi \\ 0 & \pi \leq \theta < \pi - \alpha \\ w_0 g(t + s_s[\sin\alpha x_2 - \cos\alpha x_1])\Bigl[\dfrac{s_s^2\sin^2\alpha}{(1-k_e^2)} & \\ \cdot\dfrac{\mathcal{D}_-(-\zeta_{s\alpha})\mathcal{D}_+(-\zeta_{s\alpha})}{s_{bge}^2 - s_s^2\cos^2\alpha} + \dfrac{k_e^2 s_\ell\sin\beta}{\bar{\Delta}^{(sa)}(-\zeta_{s\alpha})}\Bigr] & \pi - \alpha \leq \theta < 2\pi \end{cases} \tag{4.18}$$

where

$$\bar{\Delta}^{(sa)}(-\zeta_{s\alpha}) = s_s\sin\alpha - k_e^2 s_\ell\sin\beta \tag{4.19}$$

$$\mathcal{D}_-(-\zeta_{s\alpha})\mathcal{D}_+(-\zeta_{s\alpha}) = \frac{s_s\sin\alpha + k_e^2 s_\ell\sin\beta}{(1+k_e^2)(s_s\sin\alpha)} \tag{4.20}$$

And

$$\phi_{ra}(x_1,x_2,t) = \begin{cases} \dfrac{e_{15}}{\epsilon_{11}^s}C_f\Bigl[w_{ra}(x_1,x_2,t) + w_0 g(t - s_\ell[\sin\beta x_2 + \cos\beta x_1]) & \\ \Bigl(\dfrac{s_s^2\sin^2\alpha}{(1-k_e^2)}\dfrac{\mathcal{D}_-(-\zeta_{s\alpha})\mathcal{D}_+(-\zeta_{s\alpha})}{(s_{bge}^2 - s_s^2\cos^2\alpha)} - \dfrac{s_s\sin\alpha}{\bar{\Delta}^{(sa)}(-\zeta_{s\alpha})}\Bigr)\Bigr]\ , & 0 \leq \theta < \beta \\ \dfrac{e_{15}}{\epsilon_{11}^s}C_f w_{ra}(x_1,x_2,t)\ , & \beta \leq \theta < \alpha \\ 0\ , & \alpha \leq \theta < \pi \\ 0\ , & \pi \leq \theta < \pi - \alpha \\ \dfrac{e_{15}}{\epsilon_{11}^s}C_f w_{ra}(x_1,x_2,t)\ , & \pi - \alpha \leq \theta < \pi - \beta \\ \dfrac{e_{15}}{\epsilon_{11}^s}C_f\Bigl[w_{ra}(x_1,x_2,t) - w_0 g(t + s_\ell[\sin\beta x_2 - \cos\beta x_1]) & \\ \Bigl(\dfrac{s_s^2\sin^2\alpha}{(1-k_e^2)}\dfrac{\mathcal{D}_-(-\zeta_{s\alpha})\mathcal{D}_+(-\zeta_{s\alpha})}{(s_{bge}^2 - s_s^2\cos^2\alpha)} + \dfrac{s_s\sin\alpha}{\bar{\Delta}^{(sa)}(-\zeta_{s\alpha})}\Bigr)\Bigr]\ , & \pi - \beta \leq \theta < 2\pi \end{cases} \tag{4.21}$$

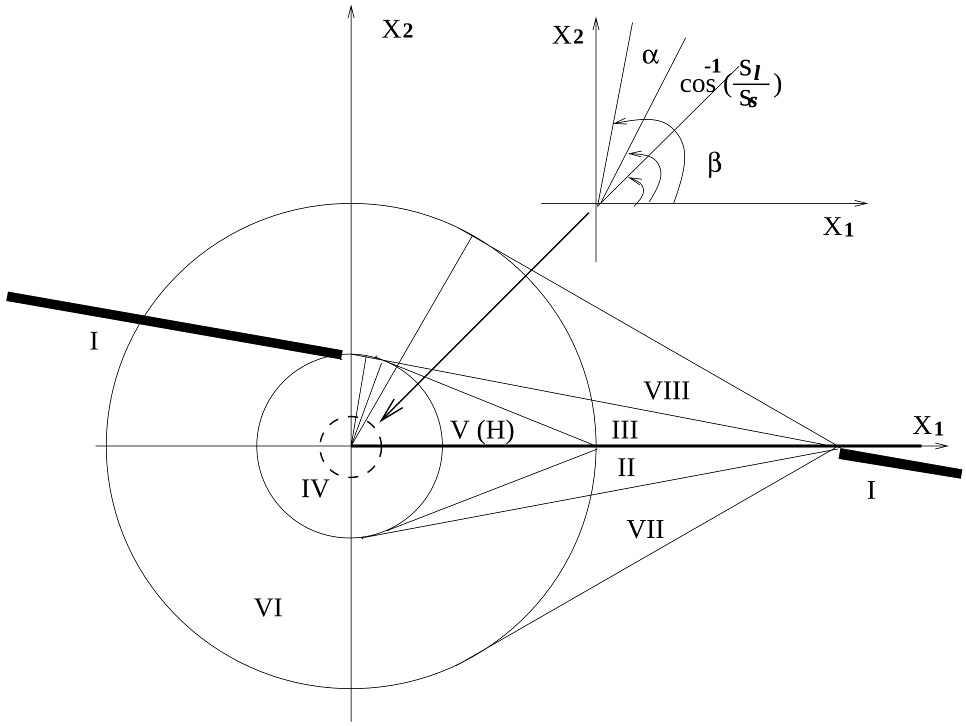

Figure 4.
The diffraction patterns excited by acoustic source (II): $\alpha \geq \cos^{-1}(s_\ell/s_s)$.

The scattering pattern in this case is far more complicated than case (1). Figure (4) shows different scattering zones in the whole plane:

I: Incident acoustic wave zone;
II: Electroacoustic wave reflection zone;
III: Electroacoustic wave refraction zone;
IV: Electroacoustic wave scattering zone;
V: Electroacoustic head wave zone;
VI: Electric wave scattering zone;
VII: Electric wave reflection zone;
VIII: Electric wave refraction zone;

Scattering fields generated by electric source

By the same token, the results of exact inversion due to electric incident source

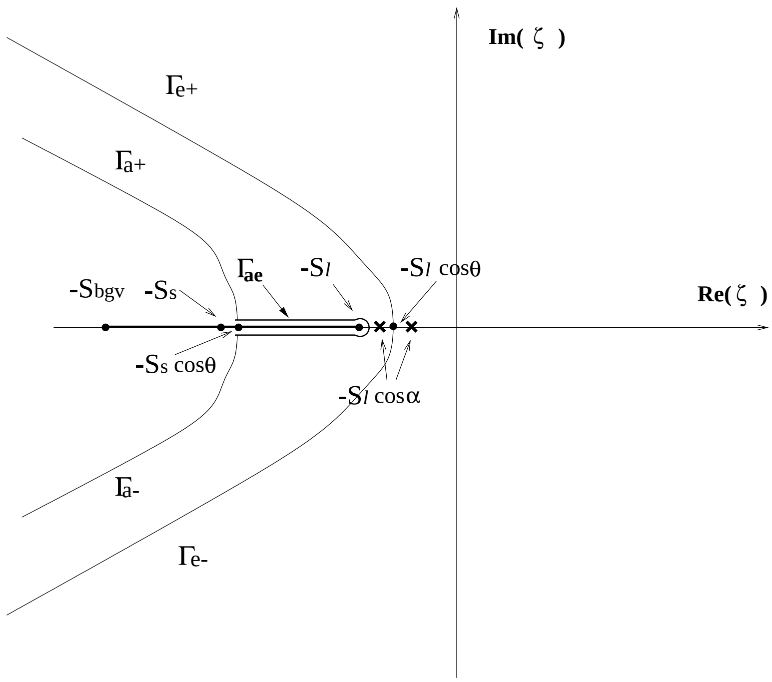

Figure 5.
Cagniard-de Hoop inversion pathes for electric excitation

can be generally expressed as

$$w^{(se)}(x_1, x_2, t) = \int_0^t g(t-\tau) w_\delta^{(se)}(x_1, x_2, \tau) d\tau + w_{re}(x_1, x_2, t) \tag{4.22}$$

$$\phi^{(se)}(x_1, x_2, t) = \int_0^t g(t-\tau) \phi_\delta^{(se)}(x_1, x_2, \tau) d\tau + \phi_{re}(x_1, x_2, t) \tag{4.23}$$

Let $\zeta_{\ell\alpha} := s_\ell \cos\alpha$. One then has

$$\begin{aligned} w_\delta^{(se)}(x_1, x_2, t) = \frac{\psi_0}{\pi} \frac{e_{15}}{\bar{c}_{44}} \Big\{ & Re\Big[\Big(\frac{a(\zeta)}{(\zeta + \zeta_{\ell\alpha}) \sqrt{t^2 - s_s^2 r^2}} \Big) \Big(- \frac{s_\ell \sin\alpha M_-(-\zeta_{\ell\alpha})}{1 - k_e^2} \frac{\sqrt{s_s + \zeta}}{s_{bge} + \zeta} \\ & \cdot \mathcal{D}_+(\zeta) \operatorname{sgn}(x_2) + \frac{a_-(-\zeta_{\ell\alpha})}{a_-(\zeta)} \frac{e_-(-\zeta_{\ell\alpha}) \bar{e}_+(\zeta)}{\bar{\Delta}^{(se)}(\zeta)} \Big) \Big] \Big|_{\zeta \in \Gamma_{a+}} H(t - s_s r) \\ & + Im\Big[\Big(\frac{s_\ell \sin\alpha M_-(-\zeta_{\ell\alpha}) a(\zeta)}{(\zeta + \zeta_{\ell\alpha}) \sqrt{s_s^2 r^2 - t^2}} \Big) \Big(\frac{\sqrt{s_s + \zeta}}{s_{bge} + \zeta} \frac{\mathcal{D}_+(\zeta)}{(1 - k_e^2)} \Big) \Big] \Big|_{\zeta \in \Gamma_{ae+}} \\ & \cdot \operatorname{sgn}(x_2) \Big(H(t - t_{ae}) - H(t - s_s r) \Big) \Big\} \end{aligned} \tag{4.24}$$

Here again only the anti-symmetry solution contributes to the electroacoustic head wave.

$$
\begin{aligned}
\phi_\delta^{(se)}(x_1,x_2,t) = \frac{\psi_0}{\pi}\Bigg\{ & Re\Big[\Big(\frac{k_e^2 a(\zeta)}{(\zeta+\zeta_{\ell\alpha})\sqrt{t^2-s_s^2r^2}}\Big)\Big(-\frac{s_\ell\sin\alpha M_-(-\zeta_{\ell\alpha})}{(1-k_e^2)}\frac{\sqrt{s_s+\zeta}}{(s_{bge}+\zeta)} \\
& \cdot\mathcal{D}_+(\zeta)\,\mathrm{sgn}(x_2)+\frac{a_-(-\zeta_{\ell\alpha})}{a_-(\zeta)}\frac{e_-(-\zeta_{\ell\alpha})\bar{e}_+(\zeta)}{\bar{\Delta}^{(se)}(\zeta)}\Big)\Big]\Big|_{\zeta\in\Gamma_{a+}} H(t-s_sr) \\
& + Im\Big[\Big(\frac{s_\ell\sin\alpha M_-(-\zeta_{\ell\alpha})a(\zeta)}{(\zeta+\zeta_{\ell\alpha})\sqrt{s_s^2r^2-t^2}}\Big)\Big(\frac{\sqrt{s_s+\zeta}}{s_{bge}+\zeta}\frac{\{\mathcal{D}_+(\zeta)\}}{(1-k_e^2)}\Big)\Big]\Big|_{\zeta\in\Gamma_{ae+}} \\
& \cdot\,\mathrm{sgn}(x_2)\Big(H(t-t_{ae})-H(t-s_sr)\Big) \\
& + Re\Big[\Big(\frac{e(\zeta)}{(\zeta+\zeta_{\ell\alpha})\sqrt{t^2-s_\ell^2r^2}}\Big)\Big(\frac{k_e^2 s_\ell\sin\alpha M_-(-\zeta_{\ell\alpha})}{1-k_e^2}\frac{\sqrt{s_s+\zeta}}{s_{bge}+\zeta} \\
& \cdot\mathcal{D}_+(\zeta)\,\mathrm{sgn}(x_2)-\frac{e_-(-\zeta_{\ell\alpha})}{e_-(\zeta)}\frac{a_-(-\zeta_{\ell\alpha})\bar{a}_+(\zeta)}{\bar{\Delta}^{(se)}(\zeta)}\Big)\Big]\Big|_{\zeta\in\Gamma_{e+}} H(t-s_\ell r)\Bigg\}
\end{aligned}
\tag{4.25}
$$

Since it is always true that $-s_\ell \le -s_\ell\cos\alpha$, $\forall\alpha\in[0,\pi/2]$ as indicated in Figure (5), there is always a simple pole, $-\zeta_{\ell\alpha} = -s_\ell\cos\alpha$, for acoustic field, thus the acoustic wave reflection/refraction zone is solely controlled by the angle of electroacoustic head wave, i.e. $\cos^{-1}\Big(s_\ell/s_s\Big)$. Define $\gamma := \cos^{-1}\Big(\tau_\ell\cos\alpha\Big)$. Apparently, $\cos^{-1}\Big(s_\ell/s_s\Big)\le\gamma\le\pi/2$. Hence, the geometrical reflection/refraction acoustic waves are

$$
w_{re}(x_1,x_2,t)=\begin{cases}
\psi_0\Big(\frac{e_{15}}{\bar{c}_{44}}\Big)g(t-s_s[\sin\gamma x_2+\cos\gamma x_1])(s_\ell\sin\alpha)\Big[-\Big(\frac{s_s\sin\gamma}{(1-k_e^2)}\Big) & \\
\frac{\mathcal{D}_-(-\zeta_{\ell\alpha})\mathcal{D}_+(-\zeta_{\ell\alpha})}{s_{bge}^2-s_\ell^2\cos^2\alpha}+\frac{1}{s_s\sin\gamma-k_e^2s_\ell\sin\alpha}\Big]\,, & 0\le\theta<\gamma \\
0\,, & \gamma\le\theta<\pi \\
0\,, & \pi\le\theta<\pi-\gamma \\
\psi_0\Big(\frac{e_{15}}{\bar{c}_{44}}\Big)g(t+s_s[\sin\gamma x_2-\cos\gamma x_1])(s_\ell\sin\alpha)\Big[\Big(\frac{s_s\sin\gamma}{1-k_e^2}\Big) & \\
\frac{\mathcal{D}_-(-\zeta_{\ell\alpha})\mathcal{D}_+(-\zeta_{\ell\alpha})}{s_{bge}^2-s_\ell^2\cos^2\alpha}+\frac{1}{s_s\sin\gamma-k_e^2s_\ell\sin\alpha}\Big]\,, & \pi-\gamma\le\theta<2\pi
\end{cases}
\tag{4.26}
$$

where

$$
\mathcal{D}_-(-\zeta_{\ell\alpha})\mathcal{D}_+(-\zeta_{\ell\alpha})=\frac{s_s\sin\gamma+k_e^2s_\ell\sin\alpha}{(1+k_e^2)s_s\sin\gamma}
$$

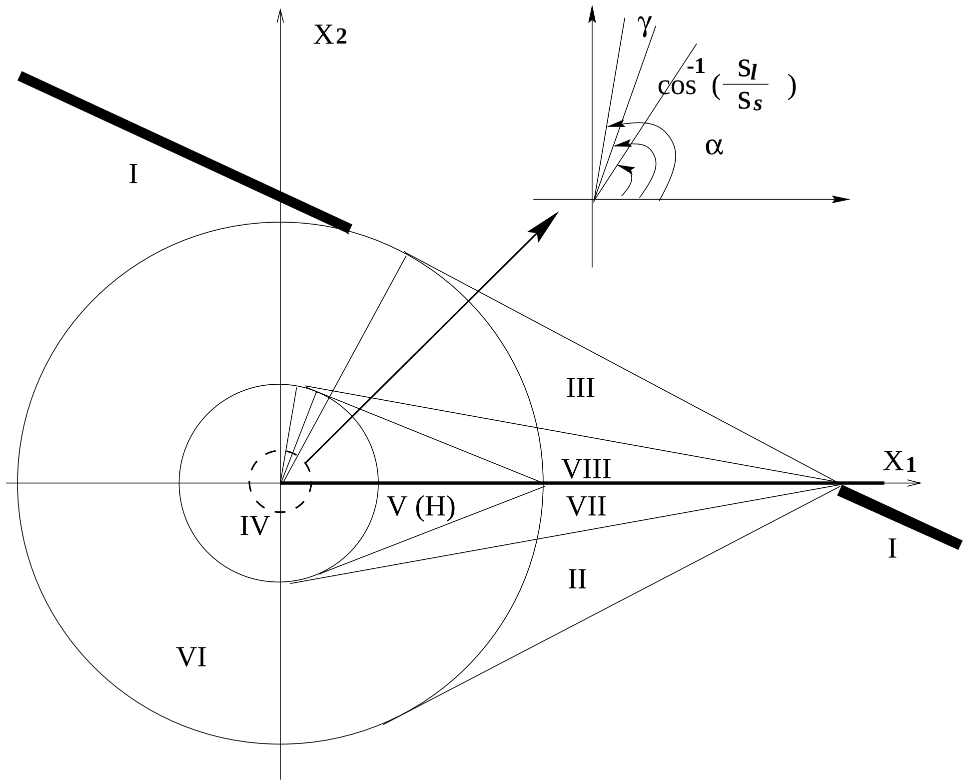

Figure 6.
Scattering fields generated by electric incident wave

On the other hand, the incident angle of the incident electric wave does affect the geometrical electric reflection/refraction waves.

$$\phi_{re}(x_1,x_2,t)=\begin{cases} \dfrac{e_{15}}{\epsilon_{11}^s}C_f w_{re}(x_1,x_2,t)+\psi_0 g(t-s_s[\sin\alpha x_2+\cos\alpha x_1])(s_s\sin\gamma) & \\ \Big[\Big(\dfrac{k_e^2 s_\ell\sin\alpha}{1-k_e^2}\Big)\dfrac{\mathcal{D}_-(-\zeta_{\ell\alpha})\mathcal{D}_+(-\zeta_{\ell\alpha})}{(s_{bge}^2-s_\ell^2\cos^2\alpha)}-\dfrac{1}{s_s\sin\gamma-k_e^2 s_\ell\sin\alpha}\Big]\,, & 0\le\theta<\alpha \\ 0\,, & \alpha\le\theta<\pi \\ 0\,, & \pi\le\theta<\pi-\alpha \\ \dfrac{e_{15}}{\epsilon_{11}^s}C_f w_{re}(x_1,x_2,t)-\psi_0 g(t+s_\ell[\sin\alpha x_2-\cos\alpha x_1])(s_s\sin\gamma) & \\ \Big[\Big(\dfrac{k_e^2 s_\ell\sin\alpha}{1-k_e^2}\Big)\dfrac{\mathcal{D}_-(-\zeta_{\ell\alpha})\mathcal{D}_+(-\zeta_{\ell\alpha})}{(s_{bge}^2-s_\ell^2\cos^2\alpha)}+\dfrac{1}{s_s\sin\gamma-k_e^2 s_\ell\sin\alpha}\Big]\,, & \pi-\alpha\le\theta<2\pi \end{cases} \tag{4.27}$$

Figure (6) illustrates diffraction pattern excited by electric incident source.

I: Incident electric wave zone;
II: Electric wave reflection zone;
III: Electric wave refraction zone;
IV: Electroacoustic wave scattering zone;
V: Electroacoustic head wave zone;
VI: Electric wave scattering zone;
VII: Electroacoustic wave reflection zone;
VIII: Electroacoustic wave refraction zone;

It may be noted that here the angle of incident wave α is within the range $0 \sim \pi/2$, which can be less or greater than the propagating angle of electroacoustic head wave angle, $\cos^{-1}(s_\ell/s_s)$, but it is always less than γ. In Figure (6), we only show the case in which $\alpha < \cos^{-1}\Big(s_\ell/s_s\Big)$.

5. Discussions

(a) Mode conversion and reflection/refraction coefficients

As shown above, acoustic incident wave can trigger electric scattering field, and vice versa; and electric incident wave can generate acoustic scattering field. It would be interesting to examine the possible mode conversion between geometrical reflection/refraction waves. To do so, similar convention used by Aki & Richards [1980a] for purely elastic wave reflection conversion and transmission is adopted here. For refraction coefficients, they are defined as

$$\begin{aligned} \acute{A}\acute{A} &:= \frac{\text{Amplitude of } w_{ra}}{\text{Amplitude of } w^{(i)}} \\ \acute{A}\acute{E} &:= \frac{\text{Amplitude of } \psi_{ra}}{\text{Amplitude of } w^{(i)}} \\ & 0 \le \theta < \pi \\ \acute{E}\acute{A} &:= \frac{\text{Amplitude of } w_{re}}{\text{Amplitude of } \psi^{(i)}} \\ \acute{E}\acute{E} &:= \frac{\text{Amplitude of } \psi_{re}}{\text{Amplitude of } \psi^{(i)}} \end{aligned} \tag{5.1}$$

Correspondingly, the refraction angles are defined as $\theta_{\acute{A}\acute{A}}, \theta_{\acute{A}\acute{E}}$, etc. .

Similarly, one can define reflection coefficients,

$$\text{ÁÀ}\ ,\ \text{ÁÈ}\ ,\ \text{ÉÀ}\ ,\ \text{ÉÈ}\ ,$$

by simply alternating the range of θ in (5.1), namely, $\pi \leq \theta < 2\pi$ and the reflection angle as well. The reflection/refraction coefficients along the slit are listed as follows: (1) $\alpha < \cos^{-1}(s_\ell/s_s)$

$$\begin{cases} \text{ÁÁ} = -\dfrac{s_s^2 \sin^2\alpha}{(1-k_e^2)(s_{bge}^2 - s_s^2\cos^2\alpha)} \\ \theta_{\text{ÁÁ}} = \alpha \end{cases} \tag{5.2}$$

$$\text{ÁÉ} = 0 \tag{5.3}$$

$$\begin{cases} \text{ÁÀ} & = \dfrac{s_s^2 \sin^2\alpha}{(1-k_e^2)(s_{bge}^2 - s_s^2\cos^2\alpha)} \\ \theta_{\text{ÁÀ}} & = -\alpha \end{cases} \tag{5.4}$$

$$\text{ÁÈ} = 0 \tag{5.5}$$

(2) $\alpha > \cos^{-1}\big(s_\ell/s_s\big)$

$$\begin{cases} \text{ÁÁ} = -\dfrac{s_s^2\sin^2\alpha}{1-k_e^2}\dfrac{\mathcal{D}_-(-\zeta_{s\alpha})\mathcal{D}_+(-\zeta_{s\alpha})}{s_{bge}^2 - s_s^2\cos^2\alpha} + \dfrac{k_e^2 s_\ell \sin\beta}{\bar{\Delta}^{(sa)}(-\zeta_{s\alpha})} \\ \theta_{\text{ÁÁ}} = \alpha \end{cases} \tag{5.6}$$

$$\begin{cases} \text{ÁÉ} = \dfrac{e_{15}}{\epsilon_{11}^s} C_f \dfrac{s_s^2\sin^2\alpha}{1-k_e^2}\dfrac{\mathcal{D}_-(-\zeta_{s\alpha})\mathcal{D}_+(-\zeta_{s\alpha})}{s_{bge}^2 - s_s^2\cos^2\alpha} - \dfrac{s_\ell \sin\beta}{\bar{\Delta}^{(sa)}(-\zeta_{s\alpha})} \\ \theta_{\text{ÁÉ}} = \beta \end{cases} \tag{5.7}$$

$$\begin{cases} \text{ÁÀ} = \dfrac{s_s^2\sin^2\alpha}{1-k_e^2}\dfrac{\mathcal{D}_-(-\zeta_{s\alpha})\mathcal{D}_+(-\zeta_{s\alpha})}{s_{bge}^2 - s_s^2\cos^2\alpha} + \dfrac{k_e^2 s_\ell \sin\beta}{\bar{\Delta}^{(sa)}(-\zeta_{s\alpha})} \\ \theta_{\text{ÁÀ}} = -\alpha \end{cases} \tag{5.8}$$

$$\begin{cases} \text{ÁÈ} = -\dfrac{e_{15}}{\epsilon_{11}^s} C_f \dfrac{s_s^2\sin^2\alpha}{1-k_e^2}\dfrac{\mathcal{D}_-(-\zeta_{s\alpha})\mathcal{D}_+(-\zeta_{s\alpha})}{s_{bge}^2 - s_s^2\cos^2\alpha} + \dfrac{s_\ell \sin\beta}{\bar{\Delta}^{(sa)}(-\zeta_{s\alpha})} \\ \theta_{\text{ÁÈ}} = -\beta \end{cases} \tag{5.9}$$

From (5.3)–(5.5) and (5.7)–(5.9), one may find that $\alpha_{cr} = \cos^{-1}\big(s_\ell/s_s\big)$ is the critical angle that controls the electric wave reflection/refraction pattern. Since

$\cos^{-1}\big(s_\ell/s_s\big) \to \pi/2$, the acoustic incident wave can induce electric wave reflection/refraction, only when the incident wave front is parallel to the slit. For the case of electric incident wave, no such distinction is necessary.

(3) $0 \le \alpha \le \pi/2$.

$$\begin{cases} \text{ÉÁ} = \dfrac{e_{15}}{\tilde{c}_{44}}\big(s_\ell \sin\alpha\big)\Big[-\dfrac{s_s\sin\gamma}{1-k_e^2}\dfrac{\mathcal{D}_-(-\zeta_{\ell\alpha})\mathcal{D}_+(-\zeta_{\ell\alpha})}{s_{bge}^2 - s_s^2\cos^2\alpha} + \dfrac{1}{\bar{\Delta}^{(se)}(-\zeta_{\ell\alpha})}\Big] \\ \theta_{\text{ÉÁ}} = \gamma \end{cases} \tag{5.10}$$

$$\begin{cases} \text{ÉÉ} \qquad = \big(s_\ell \sin\gamma\big)\Big[k_e^2\dfrac{s_s\sin\alpha}{1-k_e^2}\dfrac{\mathcal{D}_-(-\zeta_{\ell\alpha})\mathcal{D}_+(-\zeta_{\ell\alpha})}{s_{bge}^2 - s_s^2\cos^2\alpha} - \dfrac{1}{\bar{\Delta}^{(se)}(-\zeta_{\ell\alpha})}\Big] \\ \theta_{\text{ÉÉ}} = \alpha \end{cases} \tag{5.11}$$

$$\begin{cases} \text{ÉÀ} = \dfrac{e_{15}}{\tilde{c}_{44}}\big(s_\ell \sin\alpha\big)\Big[\dfrac{s_s\sin\gamma}{1-k_e^2}\dfrac{\mathcal{D}_-(-\zeta_{\ell\alpha})\mathcal{D}_+(-\zeta_{\ell\alpha})}{s_{bge}^2 - s_s^2\cos^2\alpha} + \dfrac{1}{\bar{\Delta}^{(se)}(-\zeta_{\ell\alpha})}\Big] \\ \theta_{\text{ÉÀ}} = -\gamma \end{cases} \tag{5.12}$$

$$\begin{cases} \text{ÉÈ} = -\big(s_\ell \sin\gamma\big)\Big[k_e^2\dfrac{s_s\sin\alpha}{1-k_e^2}\dfrac{\mathcal{D}_-(-\zeta_{\ell\alpha})\mathcal{D}_+(-\zeta_{\ell\alpha})}{s_{bge}^2 - s_s^2\cos^2\alpha} + \dfrac{1}{\bar{\Delta}^{(se)}(-\zeta_{\ell\alpha})}\Big] \\ \theta_{\text{ÉÈ}} = -\alpha \end{cases} \tag{5.13}$$

(b) Dynamic intensity factors

At the tail of the screen or crack, scattering fields will become singular. In what follows, the intensity factors of the singular fields generated by the antisymmetry solutions are derived.

Define

$$K_{SHT}^{(w)}(t) := \lim_{x_1\to 0^-} \sqrt{2\pi|x_1|}\sigma_{23}^{(sa)}(x_1,0,t) \tag{5.14}$$

$$K_{SHT}^{(\psi)}(t) := \lim_{x_1\to 0^-} \sqrt{2\pi|x_1|}\sigma_{23}^{(se)}(x_1,0,t) \tag{5.15}$$

$$K_{TED}^{(w)}(t) := \lim_{x_1\to 0^-} \sqrt{2\pi|x_1|}D_2^{(sa)}(x_1,0,t) \tag{5.16}$$

$$K_{TED}^{(\psi)}(t) = \lim_{x_1\to 0^-} \sqrt{2\pi|x_1|}D_2^{(se)}(x_1,0,t) \tag{5.17}$$

where the subscript "SHT" stands for stress intensity factor in a SH acoustic wave field, and subscript "TED" stands for electric displacement intensity factor in a transverse electric wave field; while the superscript "(w)" and "(ψ)" represent the sources that contribute to such intensity factors.

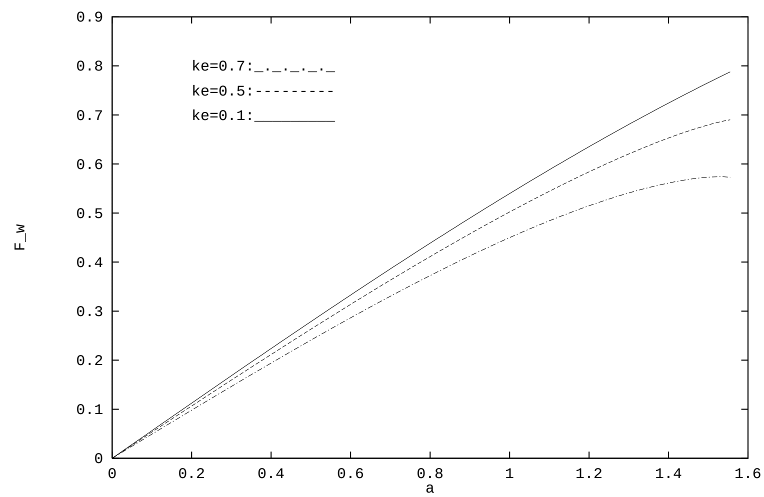

Figure 7.
Phase function $F_w(\alpha)$

Considering the asymptotic relations (Freund [1990] pp. 91-93),

$$\lim_{x_1 \to 0^-} \big(\pi |x_1|\big)^{1/2} \sigma^*_{23}(x_1, 0, p) = \lim_{\zeta \to -\infty} |p\zeta|^{1/2} \frac{\Sigma_-(\zeta, p)}{p} \tag{5.18}$$

$$\lim_{x_1 \to 0^-} \big(\pi |x_1|\big)^{1/2} D^*_2(x_1, 0, p) = \lim_{\zeta \to -\infty} |p\zeta|^{1/2} \frac{\hat{D}^*_2(\zeta, p)}{p} \tag{5.19}$$

where

$$\Sigma_-(\zeta, p) := \Sigma^{(sa)}_-(\zeta, p) + \Sigma^{(se)}_-(\zeta, p) \tag{5.20}$$

$$\hat{D}^*_2(\zeta, p) = \hat{D}^{(sa)*}_2(\zeta, p) + \hat{D}^{(se)*}_2(\zeta, p)\ . \tag{5.21}$$

and recalling (3.32(c)) and (3.81(c)), one can derive that

$$K^{(w)*}_{SHT}(p) = \sqrt{2}\tilde{c}_{44} w_0 M_-(-\zeta_{s\alpha}) s_s \sin\alpha \frac{g^*(p)}{p^{1/2}} \tag{5.22}$$

$$K^{(\psi)*}_{SHT}(p) = \sqrt{2} e_{15} \psi_0 M_-(-\zeta_{\ell\alpha}) s_\ell \sin\alpha \frac{g^*(p)}{p^{1/2}} \tag{5.23}$$

Subsequently

$$K^{(w)}_{SHT}(t) = \sqrt{\frac{2}{\pi}} \tilde{c}_{44} w_0 s_s \sin\alpha M_-(-\zeta_{s\alpha}) \chi(t) \tag{5.24}$$

$$K^{(\psi)}_{SHT}(t) = \sqrt{\frac{2}{\pi}} e_{15} \psi_0 s_\ell \sin\alpha M_-(-\zeta_{\ell\alpha}) \chi(t) \tag{5.25}$$

where

$$\chi(t) := \int_{+0}^{t} \frac{1}{\sqrt{\tau}} g(t-\tau) d\tau \tag{5.26}$$

Similarly, based on the definition,

$$\hat{D}_2^{(sa)*}(\zeta, p) = -e_{15}(1-C_f)pa(\zeta)A_{an}^{(sa)}(\zeta, p) + \epsilon_{11}^s pe(\zeta)B_{an}^{(sa)}(\zeta, p) \tag{5.27}$$

$$\hat{D}_2^{(se)*}(\zeta, p) = -e_{15}(1-C_f)pa(\zeta)A_{an}^{(se)}(\zeta, p) + \epsilon_{11}^s pe(\zeta)B_{an}^{(se)}(\zeta, p) \tag{5.28}$$

One can then show that

$$K_{TED}^{(w)}(t) = \sqrt{\frac{2}{\pi}} e_{15} \frac{s_s \sin \alpha w_0}{(1-k_e^2)} M_-(-\zeta_{s\alpha})\chi(t) \tag{5.29}$$

$$K_{TED}^{(\psi)}(t) = \sqrt{\frac{2}{\pi}} \left(\frac{{e_{15}}^2}{\tilde{c}_{44}}\right) \frac{s_\ell \sin \alpha \psi_0}{(1-k_e^2)} M_-(-\zeta_{\ell\alpha})\chi(t) \tag{5.30}$$

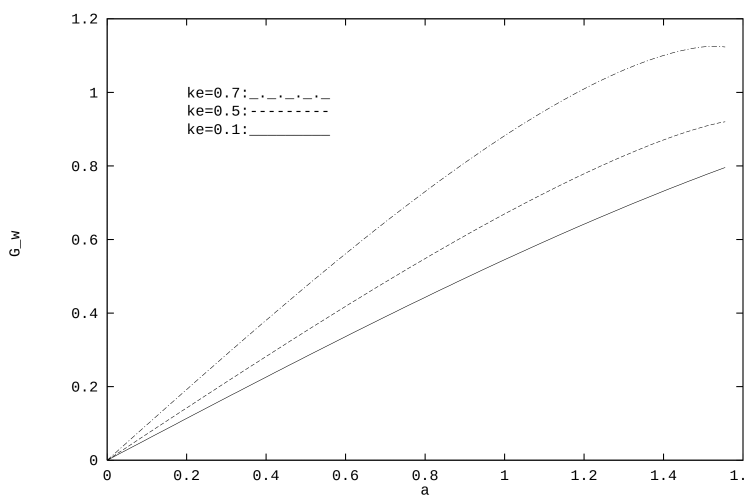

Figure 8.
Phase function $G_w(\alpha)$

The above formulas can be put into the compact forms

$$K_{SHT}^{(w)}(t) = \left(\tilde{c}_{44} w_0\right) \sqrt{s_s} F_w(\alpha) \chi(t) \tag{5.31}$$

$$K_{SHT}^{(\psi)}(t) = \left(e_{15} \psi_0\right) \sqrt{s_\ell} F_\psi(\alpha) \chi(t) \tag{5.32}$$

$$K_{TED}^{(w)}(t) = \big(e_{15} w_0\big)\sqrt{s_s} G_w(\alpha)\chi(t) \tag{5.33}$$

$$K_{TED}^{(\psi)}(t) = \big(\frac{{e_{15}}^2}{\tilde{c}_{44}}\psi_0\big)\sqrt{s_\ell} G_\psi(\alpha)\chi(t) \tag{5.34}$$

where $F_w(\alpha), F_\psi(\alpha), G_w(\alpha)$, and $G_\psi(\alpha)$ are dimensionless phase functions, which dictate the amplitudes of the intensity factors. Let $\tau_{bge} := s_{bge}/s_s$ and $\tau_\ell := s_\ell/s_s$. They can be expressed in terms of the angle of incident waves,

$$F_w(\alpha) := \sqrt{\frac{2}{\pi}}\frac{\sin\alpha\sqrt{1+\cos\alpha}}{\tau_{bge}+\cos\alpha}\Omega(\alpha) \tag{5.35}$$

$$F_\psi(\alpha) := \sqrt{\frac{2}{\pi}}\frac{\sin\alpha\sqrt{\tau_\ell}\sqrt{1+\tau_\ell\cos\alpha}}{\tau_{bge}+\tau_\ell\cos\alpha}\Xi(\alpha) \tag{5.36}$$

$$G_w(\alpha) := \sqrt{\frac{2}{\pi}}\frac{\sin\alpha}{1-k_e^2}\frac{\sqrt{1+\cos\alpha}}{\tau_{bge}+\cos\alpha}\Omega(\alpha) \tag{5.37}$$

$$G_\psi(\alpha) := \sqrt{\frac{2}{\pi}}\frac{\sin\alpha}{1-k_e^2}\frac{\sqrt{\tau_\ell}\sqrt{1+\tau_\ell\cos\alpha}}{\tau_{bge}+\tau_\ell\cos\alpha}\Xi(\alpha) \tag{5.38}$$

where

$$\Omega(\alpha) = \exp\left\{-\frac{1}{\pi}\int_{\tau_\ell}^{1}\tan^{-1}\Big[\frac{k_e^2\sqrt{(\eta-\tau_\ell)(\eta+\tau_\ell)}}{\tau_{bge}+\tau_\ell\cos\alpha}\Big]\frac{d\eta}{\eta+\cos\alpha}\right\} \tag{5.39}$$

$$\Xi(\alpha) = \exp\left\{-\frac{1}{\pi}\int_{\tau_\ell}^{1}\tan^{-1}\Big[\frac{k_e^2\sqrt{(\eta-\tau_\ell)(\eta+\tau_\ell)}}{\tau_{bge}+\tau_\ell\cos\alpha}\Big]\frac{d\eta}{\eta+\tau_\ell\cos\alpha}\right\} \tag{5.40}$$

In Figures (7) and (8), the phase functions $F_w(\alpha)$ and $G_w(\alpha)$ are plotted with different values of electro-mechanical coefficient k_e[†]. One may notice that the phase function $F_w(\alpha)$ decreases with the electro-mechanical coefficient, k_e, increases; whereas the phase function $G_w(\alpha)$ increases with the increase of electro-mechanical coefficient k_e. This can be explained as follows: when electro-mechanical coupling increases, more mechanical power is converted into electrical response, and less power remains to maintain the intensity of the mechanical field.

6. Conclusions

There are some interesting findings in this study, which, to this author's knowledge, are discovered at the first time: **1.** there is no "*shadow zone*" behind the half-plane slit; in other words, the half-plane obstacle in the piezoelectric medium is semi-transparent for both acoustic wave as well as electric wave; **2.** In mode conversion

[†] The values of phase functions $F_\psi(\alpha)$ and $G_\psi(\alpha)$ are very small.

between acoustic wave and electric wave, the critical angle of electroacoustic head wave plays an important role in controlling the occurrence of certain geometrical reflection/refraction waves. For instance, if incident acoustic wave front is almost in parallel to the half-plane, i.e. when the incident angle of acoustic source is greater than the critical angle, there will be electric reflection/refraction wave come out from the screen. **3.** Unlike the classical SH wave half-plane diffraction problem, there exists a possible bulk electric potential as well as a bulk displacement field at the tip of the slit in the piezoelectric medium.

More conclusions may be drawn by further analyzing and interpreting the analytical results presented here. As a matter of fact, in addition to this particular problem, which, this author believed, may be the most interesting one, there can be other half-plane problems in piezoelectric media, depending on how the boundary conditions are imposed. For instance, **a.** the diffraction by a rigid and perfectly conductive plate. In that case, however, both displacement as well electrical potential are zero along the screen – that leads a trivial problem in mathematical sense, because under these conditions the acoustic field and electrical field will be totally decoupled, and each yields a classical Sommerfeld solution. **b.** the diffraction by a rigid but permeable plate, by which, we mean that the total displacement is zero on the screen, but both electric displacement and the tangential electric field should be continuous across the plate. This can be coined as the de-Hoop problem [1958] in a generalized sense. **c.** probably, the most practical diffraction problem by half-plane in piezoelectric media is the diffraction problem by a semi-infinite permeable crack, i.e. a narrow semi-infinite cut filled with vacuum, or free space inside. In this case, the crack surfaces are traction free and again electric displacement and the tangential electric field should be continuous across those surfaces. Some of the solutions of above problems shall be reported elsewhere.

References

[1] Achenbach, J. D., *Wave Propagation in Elastic Solids*, North-Holland Pub. Amsterdam ·London 1973.

[2] Aki, K. and Richards, P. G., *Quantitative Seismology*, Vol. 1 & Vol. 2 W. H. Freeman, New York 1980ab.

[3] Auld, B. A., *Acoustic Fields and Waves in Solids*, Vol. 1 & Vol. 2 Second Editions Robert E Krieger Pub., Malabar, Florida 1990ab (First Edition, John & Wiley & Sons 1973ab).

[4] Bleustein, J. L., A New Surface Wave in Piezoelectric Materials. *Applied Physics Letters*, **13** (1968), 412-413.

[5] Brock, L. M. and Achenbach, J. D., Extension of an Interface Flaw under the Influence of Transient Waves *International Journal of Solids and Structures* **9** (1973), 53-67.

[6] Cagniard, L., *Reflection and Refraction of Progressive Seismic Waves*, Gauthier-Villars, Paris 1939.

[7] Freund, L. B., *Dynamic Fracture Mechanics*, Cambridge University Press 1990.

[8] de Hoop, A. T., *Representation Theorems for the Displacement in an Elastic Solid and Their Application to Elastodynamic Diffraction Theory* Doctoral Dissertation, Tech. Hogeschole, Delf 1958.

[9] de Hoop, A. T., A Modification of Cagniard's Method for Solving Seismic Pulse Problems. *Applied Scientific Research*, **B8** (1960), 349-360.
[10] Gulyaev, Y. V., Electroacoustic Surface Waves in Solids. *Soviet Physics JETP*, **9** (1969), 37-38.
[11] Jackson, J. D., *Classical Electrodynamics*, John Wiley & Sons, New York 1974.
[12] Jones, D. S., A Simplifying Technique in the Solution of a Class of diffraction Problems *Quarterly Journal of Mathematics* **3** (1952), 189-196.
[13] Jones, D. S., *The Theory of Electromagnetism*, The Macmillan Company, 1964.
[14] Li, S. and Mataga, P. A., Dynamic Crack Propagation in Piezoelectric Materials Part I: Electrode Solution. *Journal of the Mechanics and Physics of Solids*, **44** (1996a), 1799-1830.
[15] Li, S. and Mataga, P. A., Dynamic Crack Propagation in Piezoelectric Materials Part II: Vacuum Solution. *Journal of the Mechanics and Physics of Solids*, **44** (1996b), 1831-1866.
[16] Li, S., The Electromagneto-acoustic Surface Wave In A Piezoelectric Medium: The Bleustein-Gulyaev Mode. *Journal of Applied Physics*, **80** (1996), 5264-5269.
[17] Li, S., Transient Wave Propagation In Piezoelectric Half Space. *accepted for publication in ZAMP* (1998).
[18] Lin, Y. and Wang, C. and Ying, C. F., Electromagnetic Acoustic Head Waves in Piezoelectric Media. *Applied Physics Letter*, **55** (1989), 434-436.
[19] Maugin, G. A., Elastic Surface Waves with Transverse Horizontal Polarization. In: *Advances in Applied Mechanics* Edited by Yih, C. S. Vol. **23** pp. 373-434. Academic Press 1983.
[20] Miklowitz, J. *The Theory of Elastic Waves and Waveguides*, North-Holland Pub. Amsterdam · New York · Oxford 1978.
[21] Mittra, R. and Lee, S. W., *Theory of Guided Waves* The Macmillan Company, New York 1971.
[22] Noble, B., *Methods Based on the Wiener-Hopf Technique* Pergamon Press, New York 1958.
[23] Parton, V. Z. and Kudryavtsev, B. A., *Electromagnetoelasticity*, Gordon and Beach, New York 1990.
[24] Shindo, Y. and Ozawa, E., Dynamic Analysis of a Cracked Piezoelectric Material. In: *Mechanical Modelling of New Electromagnetic Materials*, editor Hsieh, R. K. T. Elsevier Pub. 1990.
[25] Sommerfeld, A., Mathematische Theorie der Diffraktion. *Mathematische Annalen* **47** (1896), 317-374.
[26] Sommerfeld, A., Theoretisches über die Beugung der Röntgenstrahlen, *Zeitschrift für Mathematik und Physik* **46** (1901), 11-97.
[27] Sommerfeld, A., Optics: Lectures on theoretical physics Vol. IV, Academic Press Inc. New York 1949.
[28] Sih, G. C. (Editor), *Mechanics of Fracture VI: Elastodynamic Crack Problems*, Noordhoff 1977.
[29] Tiersten, H. F., *Linear Piezoelectric Plate Vibrations*, Plenum Press, New York 1969.

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